Analysis of Variance
ANOVA
Proposed Schedule Changes

• Switch lecture
• No quiz
  – Informal (ungraded) presentation of term project ideas
• Read Phadke ch. 7 -- Construction Orthogonal Arrays
  – Quiz on ANOVA
  – Noise experiment due
Learning Objectives

• Introduce hypothesis testing
• Introduce ANOVA in statistic practice
• Introduce ANOVA as practiced in RD
• Compare to ANOM
• Get some practice applying ANOVA in RD
• Discuss / compare / contrast
Hypothesis Testing

A technique that uses *sample* data from a population to come to reasonable conclusions with a certain degree of confidence
Hypothesis Testing Terms

- **Null Hypothesis** ($H_0$) -- The hypothesis to be tested (accept/reject)
- **Test statistic** -- A function of the parameters of the experiment on which you base the test
- **Critical region** -- The set of values of the test statistic that lead to rejection of $H_0$
Hypothesis Testing Terms (cont.)

• **Level of significance** \((\alpha)\) -- A measure of confidence that can be placed in a result not merely being a matter of chance

• **p value** -- The smallest level of significance at which you would reject \(H_0\)

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Comparing the Variance of Two Samples

• Null Hypothesis -- $H_0: \frac{\sigma_1}{\sigma_2} = r$

• Test Statistic -- $F = \frac{1}{r^2} \frac{\text{Var}(X_1)}{\text{Var}(X_2)}$

• Acceptance criteria -- $|pF(F, d_1, d_2) - 0.5| < \frac{1 - \alpha}{2}$

• Assumes independence & normal dist.
F Distribution

- Three arguments
  - $d_1$ (numerator DOF)
  - $d_2$ (denominator DOF)
  - $x$ (cutoff)

$$F(x, d_1, d_2) = \frac{\frac{d_1}{2} \cdot \frac{d_2}{x^2} \cdot \frac{d_1}{2} \cdot \frac{d_2}{2}}{\Gamma\left(\frac{d_1}{2}\right) \cdot \Gamma\left(\frac{d_2}{2}\right)} \cdot \left[\frac{x^2 - 1}{d_1 x + d_2}\right]$$

for $x > 0$
Rolling Dice

- Population 1 -- Roll one die
- Population 2 -- Roll two die
- Go to excel sheet “dice_f_test.xls”
One-way ANOVA

- Null Hypothesis -- $H_0: \mu_1 = \mu_2 = \mu_3 = \cdots$

- Test Statistic -- $F := \frac{\text{SSB}}{\text{dfB}} \frac{\text{SSW}}{\text{dfW}}$

- Acceptance criteria -- $pF(F, \text{dfB}, \text{dfW}) < (1 - \alpha)$

- Assumes independence & normal dist.
ANOVA & Robust Design

H: This noise factor affects the mean

H: Factor setting A1 is more robust than factor setting A2

Optimize robustness

Signal Factor

Noise Factors

Product / Process

Response

Control Factors

16.881

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ANOVA and the Noise Experiment

- Did the noise factors we experimented with really have an effect on mean?
- Switch to Excel sheet “catapult_L4_static_anova.xls”
Why Test This Hypothesis?

- Factor setting PP3 is more robust than setting PP1
- Phadke -- “In Robust Design, we are not concerned with such probability statements, we use the F ratio for only qualitative understanding of the relative factor effects”
Analysis of Variance (ANOVA)

• ANOVA helps to resolve the relative magnitude of the factor effects compared to the error variance
• Are the factor effects real or just noise?
• I will cover it in Lecture 7.
• You may want to try the Mathcad “resource center” under the help menu
Additive Model

- Assume each parameter affects the response independently of the others

\[ \eta(A_i, B_j, C_k, D_i) = \mu + a_i + b_j + c_k + d_i + e \]
Analysis of Means (ANOM)

- Analyze the data to discover $m_{A1}$, $a_i$ ...
Analysis of Variance (ANOVA)

• Analyze data to understand the relative contribution of control factors compared to “error variance”

![Factor Effects on the S/N Ratio](image_url)
Breakdown of Sum Squares

GTSS

SS due to mean

SS due to factor A

SS due to factor B

Total SS

etc.

SS due to error

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Breakdown of DOF

\[ n = \text{Number of } \eta \text{ values} \]

1
SS due
to mean

\( n-1 \)

\( \text{(# levels)} - 1 \)
factor A

\( \text{(# levels)} - 1 \)
factor B

etc.

DOF for error

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Computation of Sum of Squares

- Grand total sum of squares \( GTSS = \sum_{i=1}^{n} \eta_i^2 \)
- Sum of squares due to mean = \( n\mu^2 \)
- Total sum of squares = \( \sum_{i=1}^{n} (\eta_i - \mu)^2 \)
- Sum of squares due to a factor
  = \( \text{replication#} \left[ (m_{A1} - \mu)^2 + (m_{A2} - \mu)^2 + (m_{A3} - \mu)^2 \right] \)
- Sum of squares due to error
  – Zero with no replicates
  – Estimated by “pooling”
Pooling

• Provides an estimate of error without empty columns or replicates

• Procedure
  – Select the bottom half of the factors (in terms of contribution to Total SS)
F-statistic

- Error variance = \( \frac{\text{sum of squares due to error}}{\text{degrees of freedom for error}} \)

- \( F = \frac{\text{mean square for factor}}{\text{Error variance}} \)

- mean square for factor = \( \frac{\text{SS for factor}}{\text{DOF for factor}} \)

  - F=1  Factor effect is on par with the error
  - F=2  The factor effect is marginal
  - F>4  The factor effect is substantial
Confidence Intervals for Factor Effects

• Phadke
  – Variance in $a_i$ is error variance / replication #
  – 95% confidence interval for factor effects is two standard deviations in $a_i$

• How does one interpret this value?
**Example Catapult Experiment**

- Switch to Excel “Catapult_L9_2.xls”

<table>
<thead>
<tr>
<th>A: Stop Pin</th>
<th>B: Draw Angle</th>
<th>C: Cup Position</th>
<th>D: Post Pin</th>
<th>Mean Distance</th>
<th>Std Deviation</th>
<th>Variance</th>
<th>S/N Ratio</th>
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</thead>
<tbody>
<tr>
<td>A1</td>
<td>B1</td>
<td>C1</td>
<td>D1</td>
<td>16.9</td>
<td>3.0</td>
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<td>15.1</td>
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<td>A1</td>
<td>B2</td>
<td>C2</td>
<td>D2</td>
<td>46.6</td>
<td>8.1</td>
<td>65.7</td>
<td>15.2</td>
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<tr>
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<td>B3</td>
<td>C3</td>
<td>D3</td>
<td>91.9</td>
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<td>B1</td>
<td>C2</td>
<td>D3</td>
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<td>5.8</td>
<td>34.1</td>
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<tr>
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<td>B2</td>
<td>C3</td>
<td>D1</td>
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<td>141.6</td>
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<td>C1</td>
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<table>
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<tr>
<th>GRAND MEANS</th>
<th>Mean Distance</th>
<th>Std Deviation</th>
<th>Variance</th>
<th>S/N Ratio</th>
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<tbody>
<tr>
<td></td>
<td>46.1</td>
<td>8.6</td>
<td>83.0</td>
<td>14.1</td>
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</tbody>
</table>
Homework

• Grades are exceptionally high
• Some are spending vast amounts of time
• This represents 20% of the final grade

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>101.0</th>
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<th>95.4</th>
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<td>Mean</td>
<td>94.2</td>
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<td></td>
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<tr>
<td>Standard deviation</td>
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<td>4.4</td>
<td>6.3</td>
<td>1.5</td>
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<td>98</td>
<td>109</td>
<td>101</td>
<td>97.3</td>
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Quizzes

- Some consistently score high
- Others struggling, but learning
- Remember, this is only 10%

<table>
<thead>
<tr>
<th>Quiz</th>
<th>Quiz #1</th>
<th>Quiz #2</th>
<th>Quiz #3</th>
<th>Quiz #4</th>
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<tr>
<td>Standard deviation</td>
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<td>10.4</td>
<td>22.3</td>
<td>17.6</td>
</tr>
<tr>
<td>Maximum</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>110</td>
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</tbody>
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Next Steps

• Hand in homework #5
• Homework #7 due on Lecture 10.
• Next session tomorrow
  – Present your ideas for a term project
• Following session
  – Quiz on ANOVA
  – Homework #6 (Noise Exp.) due
  – Constructing orthogonal arrays (read ch. 7)