Welcome to 16.90 iSession ...

Instructor: Turn on Webex, and distribute MuddyCards ...

Students: Please LOG OUT from your Facebook, Twitter, Google+, Foursquare, Email, Messenger, etc...
\[
\frac{\partial u}{\partial t} = - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}
\]

\[
\frac{u^{n+1} - u^n}{\Delta t} = A u^n \quad \text{explicit}
\]

\[
\frac{u^{n+1} - u^n}{\Delta t} = A u^n
\]

\[
A \text{ is finite difference } (-\frac{\partial}{\partial x} - \frac{\partial}{\partial y})
\]

\[
\left(\frac{I}{\Delta t} - A\right)u^{n+1} = \frac{u^n}{\Delta t}
\]
Scalar Conservation Laws

\[ \frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = S \]

**differential form**

\[ \int_{L}^{R} \frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} \, dx = \int_{L}^{R} S \, dx \]

\[ \Rightarrow \quad \frac{d}{dt} \int_{L}^{R} u \, dx = -F(u) \Big|_{R}^{L} + F(u) \Big|_{L}^{R} + \int_{L}^{R} S \, dx \]

**integral form**

\[ \frac{d}{dt} U = \int_{R-L}^{R-L} S \, dx \]
Characteristic Lines – Smooth Solution

Shockwaves – Shock speed

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0
\]

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u^2}{2} \right) = 0
\]
Finite Volume: Cell Average, Numerical Flux

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u^2}{2} \right) = 0
\]

\[
\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = 0 \quad \text{where} \quad F(u) = \frac{u^2}{2}
\]

\[
\bar{U}_k : = \frac{1}{\Delta X_k} \int_{L_k}^{R_k} U(x, t) \, dx
\]

size of control volume

\[
R_k - L_k
\]

uniform mesh \quad \Delta X_k = \Delta X \quad L_k = (k-1) \cdot \Delta X \quad R_k = k \cdot \Delta X
\[
\frac{d}{dt} \bar{U}_k = \frac{1}{\Delta x} \frac{d}{dt} \int_{L_k}^{R_k} u(x,t) \, dx
\]

\[
= \frac{1}{\Delta x} \left( -F \mid_{R_k} + F \mid_{L_k} + \int_{L_k}^{R_k} s \, dx \right)
\]

**Finite Volume approximation:**

\[
F \text{ at } L_k \approx F(\bar{U}_{k-1}, \bar{U}_k)
\]

\[
F \text{ at } R_k \approx F(\bar{U}_k, \bar{U}_{k+1})
\]
First Order Upwind Scheme

\[ F_{k+\frac{1}{2}} := F \bigg|_{R_k} = F \bigg|_{L_{k+1}} \]

\[ F_{k+\frac{1}{2}} = \begin{cases} 
F(\bar{u}_k) = \frac{\bar{u}_k}{2} & \text{if } \frac{\bar{u}_{k+1} + \bar{u}_k}{2} > 0 \\
F(\bar{u}_{k+1}) = \frac{\bar{u}_{k+1}}{2} & \text{else}
\end{cases} \]
Shock capturing of Finite Volume