Welcome to 16.90 iSession ...

Instructor: Turn on Webex, and distribute MuddyCards ...

Students: Please LOG OUT from your Facebook, Twitter, Google+, Foursquare, Email, Messenger, ...etc... ...etc... ...etc...
Shock capturing of Finite Volume

\[ \int \frac{\partial p}{\partial t} + \nabla \cdot F(p) = 0 \]

differential form

\[ \frac{\partial p}{\partial t} + \frac{\partial F(p)}{\partial x} = 0 \]

\[ \frac{d}{dt} \int_{\Omega} p \, dx + \int_{\Omega} \hat{n} \cdot F(p) = 0 \]

integral form

\[ \frac{d}{dt} \int_{L} p \, dx + F(p) \bigg|_{r} - F(p) \bigg|_{l} = 0 \]
Upwind Scheme for finite volume

\[ F_{k-\frac{1}{2}} = \begin{cases} F(P_{k-1}) & \text{if } c > 0 \\ F(P_k) & \text{if } c < 0 \end{cases} \]

\[ C : \quad \frac{\partial p}{\partial t} + c \frac{\partial p}{\partial x} = 0 \]

\[ C = \frac{\Delta F}{\Delta p} \]

\[ \frac{\partial p}{\partial t} + \frac{\partial F(p)}{\partial x} = 0 \]
Why Upwind Scheme?
Traffic jam simulation

\[ \rho(x,t) \]
\[ \rho = 0 \quad \text{empty} \]
\[ \rho = 1 \rightarrow \]
\[ u(\rho) = 0 \rightarrow \]
\[ u(\rho) = 1 : 85 \text{ mph} \]
\[ U(\rho) = 1 - \rho \]

What's \( F(\rho) = \rho \cdot u = \rho - \rho^2 \)
\[ \frac{\partial \rho}{\partial t} + \frac{\partial F(\rho)}{\partial x} = 0 \]

where \( F(\rho) = \rho - \rho^2 \)

\[ C = 1 - 2\rho = \frac{dF}{d\rho} \]

(highly)