Welcome to 16.90 iSession ...

Instructor: Turn on Webex, and distribute MuddyCards ...

Students: Please LOG OUT from your Facebook, Twitter, Google+, Foursquare, Email, Messenger, ...etc... ...etc... ...etc...
Review and outlook

- Local accuracy (including consistency)
- Global accuracy
- Zero stability
- Eigenvalue stability

\[ \frac{d\mathbf{u}}{dt} = \lambda \mathbf{u} \]

- Newton Raphson iteration
Eigenvalue stability analysis


\[
\frac{V^{n+1} - V^n}{\Delta t} = f(V^n) = \lambda V^n
\]

\[
\frac{V^{n+1}}{\Delta t} = \frac{V^n}{\Delta t} + \lambda V^n
\]

\[
V^{n+1} = \left(1 + \Delta t \lambda\right) V^n
\]
Eigenvalue stability analysis


- Discuss stability for Forward Euler, Backwards Euler and Midpoint Rule, for the ODE:

\[
\frac{dx}{dt} = \lambda x \quad \gamma = e^{\lambda t}
\]

where \(\lambda\) can take any complex value

- For what \(\lambda\) and \(\Delta t\) is each scheme stable?
Eigenvalue stability analysis


• Discuss stability for Forward Euler, Backwards Euler and Midpoint Rule, for the coupled ODEs:

\[
\frac{dx}{dt} = -y - \epsilon x
\]

\[
\frac{dy}{dt} = x
\]

where \( \epsilon \) range from 0 to \( \infty \)

• For what \( \epsilon \) and \( \Delta t \) is each scheme stable?
\[
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]

\[V^{n+1} = V^{n-1} + 2\Delta t f(V^n)\]

\[\frac{dy}{dt} = \lambda y\]

\[V^{(n+1)} = V^{(n-1)} + 2\Delta t \lambda V^n\]

\[V^{(n+1)} = V^{(0)} z^{n+1} \quad V^{(n)} = V^{(0)} z^n \quad V^{(n-1)} = V^{(0)} z^{n-1}\]

\[z^{n+1} = z^{n-1} + 2\Delta t \lambda z^n\]

\[z^2 = 1 + 2\Delta t \lambda z\]

\[z = \frac{2\lambda \Delta t \pm \sqrt{4\lambda^2 \Delta t^2 + 4}}{2}\]
\[ z = e^{i\theta} \]

\[ e^{2i\theta} = 1 + 2(a + \lambda) e^{i\theta} \]
Newton Raphson for implicit scheme

\[ \frac{du}{dt} = -u^2 \]

\[ u(0) = 1 \]

\[ u \to u + V \text{ where } V \text{ is very small} \]

\[ \frac{d(u+V)}{dt} = - (u+V)^2 \]

\[ \frac{du}{dt} + \frac{dv}{dt} = -u^2 - 2uv \text{ (very very small)} \]

\[ \frac{dv}{dt} = -2uv \]
When is FE stable?

\[ U(t) = \frac{1}{t+1} \]

\[ -2 \leq -2ue^t \leq 0 \quad \text{for } FE. \]

\[ \frac{dy}{dt} = -U^2 \quad \text{BE?} \]

\[ \frac{U^{n+1} - U^n}{\Delta t} = f(U^n) \quad \text{Forward Euler} \]

\[ f(U^n) = -(U^{n+1})^2 \quad \text{Backward Euler} \]
\[ \frac{u^{n+1} - u^n}{\Delta t} = -\left( u^{n+1} \right)^2 \]