Analysis of Historic Structures

Lecture 2: Intro to Masonry Structure
Review of Last Meeting

- Principles of historic structure
  - EQUILIBRIUM

- Lower bound and upper bound theorems

- Possible research topics
Lower Bound Theorem
Upper Bound Theorem

\( \lambda \mathcal{P} \)
Theorems of Limit Analysis

1. **Lower Bound**: If you can demonstrate at least one possible equilibrium state, then the structure can also find at least one possible stable state.

2. **Upper Bound**: When the load path can no longer be contained within the structure, and it is the unique and largest possible load, then it is the collapse load.
Hooke’s “2nd” Law (1675)

“ut pendet continuum flexile, sic stabit contiguum rigidum inversum”

As hangs the flexible line, so but inverted will stand the rigid arch.
Compression vs. Tension

Compression

Tension
Equilibrium at a Point

\[ \Sigma F = 0 \] (sum of forces is zero)
Structural Equations

Only three types of equations:

1) Equilibrium
2) Material properties (elasticity, etc)
3) Compatibility (geometry)

We will focus on equilibrium equations because they are the most important.
Graphic Statics

Applet by Simon Greenwold
Hooke’s Hanging Chain
Design of Masonry

- Main principle: must be kept in compression

- Also applies to cast iron, unreinforced concrete, and other “brittle” materials
3D Vaults: “Slice” into arches

(a) Collapse of dome and drum in "orange-slice" segments
Structural Analysis of Masonry

- **The Stone Skeleton**
  by Jacques Heyman

- Three main assumptions:
  - No tensile strength
  - Infinite compressive strength (rigid)
  - Sliding does not occur
Arch on Spreading Supports
Range of Arch Thrust

Internal thrust lines due to self weight of arch
Range of Arch Thrust

TENSION

\( H_{\text{max}} \)

\( H_{\text{min}} \)
Range of Arch Thrust

COMPRESSION
Model Arch Experiment
Model Arch at Collapse State
Understanding cracks in masonry

1. Why do cracks occur?
2. What do they tell us?
3. Are they a cause for concern?
Understanding cracks in masonry

1. Why do cracks occur?
   - Small movements of supports

2. What do they tell us?
   - Where forces are NOT acting

3. Are they a cause for concern?
   - Usually not, but they can be
Understanding of Collapse

Causes of collapse:

1. Displacements
   - Foundation movements, mortar “creep” over time

2. Overloading (truck on a bridge)
   - Water on vaults, collapsing roof on vault

3. Accelerations
   - Vibrations, earthquakes
Stability rather than failure of the material is the dominant concern

Collapse occurs when the load path can no longer be contained within the masonry
Model Arch at Collapse State
Single Span Stone Arch
Single Span Arch
Single Span Arch
Single Span Arch
Double Span Stone Arch
Double Span Arch with Fill
Double Span Arch with Fill
Double Span Arch with Fill
• This makes sense for bridges, but buildings don’t usually have trucks driving on top of the vaults

• Deformation over time can cause collapse in buildings
Conclusions

- Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety

- Determine collapse states based on thrust line analysis using graphic statics

- Equilibrium equations are most important when analyzing historical structures