Lecture 3

Analysis of Masonry Structures: Arches, Vaults, and Buttresses
Limit Analysis of Masonry

- **Lower Bound Theorem**
  - Seeks permissible line of compressive force for the given loading

- **Upper Bound Theorem**
  - Seeks critical load which results in a failure mechanism
Poleni (1748) applies lower bound to St. Peter’s of Rome
Selby Abbey, 12th C, England

- Tower construction “punches” through
- Arches deform to accommodate support movements
- Stable because a line of thrust can be found within the deformed arch
Experiments by Danyzy (1732)

Collapse occurs by hinging between blocks, when a load path can not be contained within the masonry.

Safety is a question of geometry and stability, not crushing of stone.
Cathedral in Palma de Mallorca, Analysis by Joan Rubio (1912)
Analysis of a Flat Arch

Image courtesy of Denis Y. Yu, structurae.de
Analysis of a Flat Arch

Thrust of a flat arch?
Safety of an Arch on Buttresses

Must solve three problems:

1. Load capacity of buttress (and influence of lean)
2. Collapse state of arch on spreading supports
3. Analysis of arch supported on leaning buttresses
Buttress Analysis, DuPuit (1870)

Fig. 86.

Fig. 87.
Heyman on Leaning Walls (1992)
Buttress Collapse

Buttress Collapse

Possible fracture

Locus of pressure points

Wb

Wb-V

Wb+V

Wb+V

Wb+V-Wc

(a) (b) (c)

Hs

Hu

V

m

n

p
Assumed Compressive Stress Distribution

- Interface between stones
- Linear compressive stress distribution
- Locus of pressure points (line of internal forces)
- Opening between stones
- Ineffective area
- $m$ and $2m$
Determine Shape of Fracture
Assumed Buttress Stress Distribution at Collapse

A-A: Locus of pressure points acts near the centroid and the entire section is in compression.

B-B: Locus of pressure points reaches kern point of rectangular section (1/3 point).

C-C: Section properties change as fracture occurs and locus moves to 1/3 of the new section.

Fracture reduces thrust capacity by >30% in many cases
Vicat Experiments on Suspension Bridge Towers (1832)

Experiences sur des piliers à parois verticaux.

A. 1° avec des attaches enveloppantes. Fig. 1.
Vicat Experiments on Suspension Bridge Towers (1832)
Model Buttress Experiments

Fracture reduces thrust capacity by 20% to 30%
Gothic Buttress Failure State

(a) $V \leftarrow H_a$

(b) $V \leftarrow H_b$
Leaning Buttress

(a) $W_b$ - $W_c + V$
(b) $W_b - W_c$
(c) $W_b - 2W_c + V$
Leaning Buttress Capacity

Angle of lean, $\phi$ (radians)

Horizontal thrust capacity of buttress, $H_{\phi}$

Vertical buttress

$bh_0\gamma\left(\psi + \frac{1}{2\mu}\right)$

Maximum lean
Load Capacity of a Buttress

- A masonry buttress will fracture at collapse, reducing its load capacity.

- A leaning buttress has a linear reduction in capacity, based on a small angle approximation as the centroid shifts horizontally.
Church in Guimarei, Spain

Geometry changes may threaten stability of the structure

Huerta and Lopez (1997)
Church in Guimarei, Spain

1. Buttress leans outward (e.g. foundation deforms)
2. Arch deforms and thrust increases
3. Buttress leans further and thrust increases further.
Church in Guimarei, Spain

Must solve three problems:

1. Load capacity of buttress (and influence of lean)
2. Collapse state of arch on spreading supports
3. Analysis of arch supported on leaning buttresses
Spreading Arches:
Viollet’s study of Vezelay (1854)
Viollet-le-Duc on Spreading Arches
Arch on Spreading Supports
Flying Buttresses at Palma de Mallorca
Conclusions

- Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety

- Limit analysis can be used to determine collapse states based on thrust line analysis

- Capacity for displacements may be more important than load capacity (particularly for historic buildings)

- For high vaulted buildings, the arch will collapse and the buttress will remain standing in most cases.
Research Papers on Masonry

- Comparative studies
  - Arches
  - Vaulting
  - Buttresses
  - Individual structures
Research Papers on Masonry

- Tile vaulting (Guastavino)
- Gothic
- Romanesque
- Mamluk
- Maya/Aztec
- Mycenaean tholos tombs
- Individual structures