1.) (20 points) DNA sequence

...GTAGCCGTGAATCGATGGTCCAAATACCG...
|----------?>

You have a genome that is exactly $1 \times 10^9$ bases long. You would like to choose a DNA sequence out of the genome that is unique. What is the minimum length of the sequence such that it could be unique (i.e., it is possible that it does not occur anywhere else in the genome)?
2.) The protein below has four binding sites ($\alpha$, $\beta$, $\gamma$, $\delta$) for the ligand L. We would like to find its equilibrium binding population. For now assume that the association and dissociation constants are equal.

\[
\begin{align*}
+ \text{L} & \quad \rightarrow \quad \text{L} \\
\end{align*}
\]

a) (15 points) Calculate $W$ and the entropy (in units of k) for the situation in which

i) 0 ligands are bound ($N_L = 0$)

ii) 1 ligand is bound ($N_L = 1$)

iii) 2 ligands are bound ($N_L = 2$)

iv) 3 ligands are bound ($N_L = 3$)

v) 4 ligands are bound ($N_L = 4$)
b) (5 points) Which states have the **highest** entropy?

c) (5 points) Which states have the **lowest** entropy?

d) (5 points) Which state will you find it in at equilibrium?

e) (20 points) Let’s say that the binding constants are not equal, i.e., ligand L has a higher probability of being bound:

\[ p_{\text{bound}} = 0.75 \]
\[ p_{\text{unbound}} = 0.25 \]

Plot the probability distribution for all of the states, \( p(i) \).
3.) DNA has 3 different possible configurations: A, B, and Z:

Let the scores for each configuration $\epsilon_i$ be: $\epsilon_A = 1$, $\epsilon_B = 2$, $\epsilon_Z = 3$.

a) (10 points) Write down the partition function, $q$. (you may substitute $x = e^\beta$)

b) (10 points) Write down the expressions for $p_A$ and $p_B$ and $p_Z$.

c) (10 points) What is the variance of the distribution, $\langle \sigma^2 \rangle$, if all three configurations are equally possible?