Key Concepts for section IV (Electrokinetics and Forces)

1: Debye layer, Zeta potential, Electrokinetics
2: Electrophoresis, Electroosmosis
3: Dielectrophoresis
4: Inter-Debye layer force, Van-Der Waals forces
5: Coupled systems, Scaling, Dimensionless Number

Goals of Part IV:

(1) Understand electrokinetic phenomena and apply them in (natural or artificial) biosystems

(2) Understand various driving forces and be able to identify dominating forces in coupled systems
Helmholtz model (1853)

Guoy-Chapman model (1910-1913)

Stern model (1924)

slip boundary

$\Phi_0$

$\kappa^{-1}$

$X$

$\kappa^{-1}$

$X$

$\kappa^{-1}$

$X$

$\xi$ (zeta potential)
Electroosmosis

- The oxide or glass surface become unprotonated (pK ~ 2) when they are in contact with water, forming electrical double layer.

- When applied an electric field, a part of the ion cloud near the surface can move along the electric field.

- The motion of ions at the boundary of the channel induces bulk flow by viscous drag.

Figure by MIT OCW.
Fick’s law of diffusion

Concentration ($c$) ($\rho$)

Osmosis

Convection

Maxwell’s equation

E and B field

$\rho_e, J_e : \text{source}$

Electrophoresis

Electroosmosis

Streaming potential

(aqueous) medium, Flow velocity ($v_m$)

Navier-Stokes’ equation
Slip boundary, zeta potential

Stern layer: adsorbed ions, linear potential drop
Gouy-Chapman layer: diffuse-double layer, exponential drop
Shear boundary: \( v_z = 0 \)

Navier-Stokes equation

\[
\rho \frac{d\vec{v}}{dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho_e \vec{E} \cong 0
\]

\[
\nabla \cdot \vec{v} = 0 \text{ (incompressible)}
\]
\[ v_z(r) = -\frac{\varepsilon (\zeta - \Phi(r))}{\mu} E_{zo} - \left(\frac{R^2 - r^2}{4\mu}\right) \frac{\Delta P}{L} \]

**Poiseuille flow**

\( \Delta P \neq 0, E_z = 0 : \) Poiseuille flow
parabolic flow profile

**Electroosmotic flow**

\( \Delta P = 0, E_z \neq 0 : \) Electroosmotic flow
flat (plug-like) profile

\[ v_{EEO} = -\frac{\varepsilon \zeta}{\mu} E_z = \mu_{EEO} E_z \quad (outside \ of \ the \ Debye \ layer) \]

\( \mu_{EEO} : \) electroosmotic 'mobility'
Electrophoresis: Simple model?

\[ \vec{F}_{el} = q \vec{E}_e \]

\[ \vec{F}_{drag} = 6\pi R \mu \nu_{ep} \]

\[ \vec{F}_{net} = \vec{F}_{el} - \vec{F}_{drag} = q \vec{E} - 6\pi R \mu \nu_{ep} = 0 \]

\[ \therefore \quad \frac{\nu_{ep}}{E_z} = u_{ep} = \frac{q}{6\pi R \mu} \]

This is wrong!
Electrophoresis: real picture

\[ \vec{v}_{ep} = u_{ep} \vec{E} \]

\( \vec{E} \)

\( \mu_{ep} \) is a complex, electromechanically coupled process.
- E field is distorted around the particle.
- Counterions are moving in the opposite direction.
- Fluid slip (friction) is localized within the Debye layer.
Limiting case: $\kappa R >> 1$ (particle size $>>$ Debye layer thickness)

High ionic strength (high buffer concentration) condition
Electromechanical coupling (friction) happens within the Debye layer.

Fluid at rest  "electrophoresis"

particle at rest  "electroosmosis"
Similarity to electroosmosis

At small Debye length, surface curvature doesn’t matter. Situation similar to electroosmosis at planar surface. Friction due to the particle motion occurs mostly within the Debye layer. Outside of the Debye layer: no fluid flow gradient (electroneutral)
Proteins: **3D structure** with complex charge distribution

Human Serum Albumin

DNA (SDS-proteins): **Linear polymer** with uniform charge density

DNA


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Polyelectrolyte electrophoresis: Free-draining

- When driven by an electric field
- DNA and counterions are dragged in the opposite direction
- Hydrodynamic interaction screened
- Friction with solvents occurs at every monomers
  \[ \zeta_{\text{friction}} \sim N \]

- When driven by a hydrodynamic pressure
- DNA and solvent molecules are moving together
- Hydrodynamic interaction keeps the blob move together
- Friction with solvents occurs at the surface of the blob
  \[ \zeta_{\text{friction}} \sim 6\pi \eta R \sim N^v \]
DNA Sequencers

**Slab gel sequencer**

**Multiple capillary sequencer**

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**Fig. 1—“Slab gel” DNA Sequencer.**
Conventional DNA sequencer uses slab gel. Separation gel is formed in the gap between two flat glass plates.

**Fig. 2—Multi-capillary DNA Sequencer.**
Samples in sample tray are loaded into capillaries automatically. DNA fragments separated through the capillary by electrophoresis are detected by laser-induced fluorescent detection.

Courtesy of Hitachi Review. Used with permission.

From Hitachi Review Vol. 48, No. 3, 107 (1999), Kazumichi Imai, Satoshi Takahashi, Masao Kamahori, Yoshinobu Kohara
Micro Total Analysis System (microTAS): Parallelism

- 96~356 samples analyzed in a single chip simultaneously
- fluorescence detection of DNA at the center of the chip (rotating optical head)

Figure 1 removed due to copyright restrictions.

Yining Shi et al., Analytical Chemistry, 71, 5354 (1999)
Micro Total Analysis System (microTAS): Integrability

Images removed due to copyright restrictions.

M. Burns et al., Science, 282, 484 (1998)
Technology Need for Advanced Biosensing

- **Challenges of Sample Complexity**
  - Blood serum / Urine / Saliva
  - Highly diverse: more than ~10,000
  - 90% of total serum protein: albumin and globulin (~mg/ml level)
  - biomarkers and cytokines: 10ng/ml or less (up to $10^9$ dynamic range)
Electrophoresis is a *complicated* electrokinetic phenomena.

(determined by zeta potential, not the net charge of the molecule)
Three images removed due to copyright restrictions. 
Source: Alberts et al., Molecular Biology of the Cell.

- Slab Gel electrophoresis (Length-based Separation): see Figure 4-42.
- Isoelectric focusing (charge-based separation): see Figure 4-44.
- 2D protein separation: see Figure 4-45.