Problem 1: Prob 6.1a in FFF, Cylindrical pore model: electrokinetic transduction

Problem 2: Flow Separation Chip

A continuous-flow cell-separation chip is shown schematically in the figure below (top view). The objective is to separate normal cells from cells that have had their glycocalyx surface charge groups modified by means of antibodies bound to the surface glycoproteins. (The antibody binding is used to identify specific glycoproteins on the cell surface.) Cells are injected into the channel at x = 0, and electrodes at the ends of the channel are used to apply an electric field in the x-direction E_x = 1,000 V/m. An optical detector is situated at x = 1 cm downstream in the channel which can sense both normal and modified cells.

(NOTE: in reality, microfabrication enables the channel to be patterned in a winding fashion so that the chip is quite small; but the effective channel length is 1 cm, and you can assume that the electric field can be treated as uniform within the winding channel.)

While channels in microfluidics devices typically have rectangular cross-sections, for the purposes of this problem, assume that the channel is cylindrical with radius R = 100 µm.

(a) Find an analytical expression for the mean velocity, U, of the fluid in the channel after the voltage is applied. Show your reasoning for all steps. Calculate U using the parameter values given and other appropriate constants.

(b) Find expressions for the velocity (with respect to the stationary lab frame) of the normal and modified cells v_n and v_m respectively. Show your reasoning for all steps. Calculate v_n and v_m. How long does it take for the normal and modified cells to begin to reach the detector?
(c) Does $U$, $v_n$, and $v_m$ change significantly for the case of a hypotonic buffer, e.g., $10^{-5}$ M NaCl (which can be accomplished without killing the cells by using sucrose to maintain normal osmolarity)?

![Diagram of a channel with normal and modified cells]

- **Normal Cell**
- **Modified Cell**

- Total channel length = 1 cm
- Channel radius = 100 $\mu$m
- Buffer $c_o = 0.1$M NaCl
- Zeta ($\zeta$) potential (channel wall) = -50 mV
  - ($\zeta$) (normal cell membrane) = -20 mV
  - ($\zeta$) (modified cell membrane) = + 40 mV
- Cell radius = 10 $\mu$m