PROBLEM SET 4. Bayesian Networks, Refining Protein Structures in PyRosetta, Mutual information of protein residues (21 Points)

Due: Thursday, April 17th at noon.

Python Scripts
All Python scripts must work on athena using /usr/athena/bin/python. You may not assume availability of any third party modules unless you are explicitly instructed so. You are advised to test your code on Athena before submitting. Please only modify the code between the indicated bounds, with the exception of adding your name at the top, and remove any print statements that you added before submission.

Electronic submissions are subject to the same late homework policy as outlined in the syllabus and submission times are assessed according to the server clock. Any Python programs you add code to must be submitted electronically, as .py files on the course website using appropriate filename for the scripts as indicated in the problem set or in the skeleton scripts provided on course website.
You are given two different Bayesian network structures 1 and 2, each consisting of 5 binary random variables A, B, C, D, E. Each variable corresponds to a gene, whose expression can be either “ON” or “OFF”.

(A – 2 points) In class, we covered the chain rule of probability for Bayes Nets, which allows us to factor the joint probability over all the variables into terms of conditional probabilities. For each of the following cases, factor P(A,B,C,D,E) according to the independencies specified and give the minimum number of parameters required to fully specify the distribution.

(i) A,B,C,D,E are all mutually independent

(ii) A,B,C,D,E follow the independence assumptions of Network #1 above

(iii) A,B,C,D,E follow the independence assumptions of Network #2 above

(iv) no independencies
(B – 3 points) Using Network #2 and the probabilities given below, calculate the probability of the following:

\[
P(A = ON) = 0.6
\]

\[
P(B = ON \mid A) = \begin{cases} 
0.1, & A = OFF \\
0.95, & A = ON 
\end{cases}
\]

\[
P(C = ON \mid A) = \begin{cases} 
0.8, & A = OFF \\
0.5, & A = ON 
\end{cases}
\]

\[
P(D = ON \mid A, B) = \begin{cases} 
0.1, & A = OFF, B = OFF \\
0.9, & A = ON, B = OFF \\
0.3, & A = OFF, B = ON \\
0.95, & A = ON, B = ON 
\end{cases}
\]

\[
P(E = ON \mid D) = \begin{cases} 
0.8, & D = OFF \\
0.1, & D = ON 
\end{cases}
\]

(i) \( P(A=ON, B=ON, C=ON, D=ON, E=ON) \)

(ii) \( P(E = ON \mid A = ON) \)
(iii) $P(A = \text{ON} \mid E = \text{ON})$
(C – 1 point) For the rest of this problem, you will be using the Python module Pebl
(https://code.google.com/p/pebl-project/), which provides an environment for learning the
structure of a Bayesian network. Just like PyRosetta, Pebl has been installed on Athena, and
we will provide instructions for how to complete this problem on Athena’s Dialup Service.
You are, of course, free to download Pebl yourself and complete the problem locally.

Log on to Athena’s Dialup Service:

ssh <your Kerberos username>@athena.dialup.mit.edu

Before running any Python scripts, use the following command to add the Athena Pebl
module to your PYTHONPATH:

```
export PYTHONPATH=/afs/athena/course/20/20.320/pythonlib/lib/python2.7/site-packages/
```

If you forget to do this, you will get
`ImportError: No module named pebl`
when you try to run the provided scripts. Additional information about Pebl and its modules
can be found here: https://pythonhosted.org/pebl/apiref.html#apiref.

You will also need to get the .zip containing the files for this problem in the course folder:

```
cd /afs/athena/course/7/7.91/sp_2014
cp bayesNetworks.zip ~
```

```
cd ~
unzip bayesNetworks.zip
```
```
cd bayesNetworks
```

We have provided you with a small subset of the microarray data that were published in a
explored gene expression changes in S. cerevisiae in response to a variety of environmental
stresses, such as heat shock and oxidative stress. We have given you geneExprData.txt, which contains data for 12 of the genes included in the study, observed under these various stress conditions.

We have also provided a simple script learnNetwork.py which will learn the network structure
that best explains these data, using Pebl’s greedy learner algorithm (see
https://pythonhosted.org/pebl/learner/greedy.html). From the folder containing
geneExprData.txt, run the script:

```
python learnNetwork.py geneExprData.txt network1
```

If you’ve done this correctly, a new folder called network1 will be created in your current
directory on Athena, containing an .html file and two more folders data and lib. If you are at
an Athena workstation, you can open and view the .html directly. If you’ve ssh’ed into
Athena from your own computer, you will need to copy the entire outFolderName to your local
computer using scp:
<in a new Terminal on your computer, cd into your local computer’s directory where you want
to download the files>
scp -r <your Kerberos username>@athena.dialup.mit.edu:/bayesNetworks/network1 .

Now click on the .html file to open it. Include a printout of the top scoring network with your
write-up or upload a photo of it to the course website online dropbox. What is its log score?

(D – 1 point) We have provided another data file, exprDataMisTCP1.txt, which is the same
as geneExprData.txt except the data for tcp1 has been removed.

(i) Before using Pebl to calculate the network, make a quick guess about how the network
might change in response to removing the tcp1 data.

(ii) Now, use the learnNetwork.py script to learn a network for exprDataMisTCP1.txt:

python learnNetwork.py exprDataMisTCP1.txt network2

Again, if you ssh’ed into Athena, you will need to copy the network2 folder locally in order to
view it:

<in a new Terminal on your computer, cd into your local computer’s directory where you want
to download the PDF>
scp -r <your Kerberos username>@athena.dialup.mit.edu:/bayesNetworks/network2 .

Include a printout of the top scoring network with your write-up or upload a photo of it to the
course website online dropbox. According to this network, which node(s) does the expression
of mcx1 depend on? Is this consistent with your guess above? Briefly suggest a reason why
you might be observing this network in response to loss of tcp1 data.
P2 – Refining Protein Structures in PyRosetta (7 points)

In this problem, you will explore refining protein structures using two methods discussed in class: Energy Minimization and Simulated Annealing. To do this, we will use PyRosetta, an interactive Python-based interface to the powerful Rosetta molecular modeling suite.

When implementing your solutions in the skeleton code provided, we encourage you to look at the solutions from Problem Set 3 Question 3 as well as the PyRosetta documentation: http://graylab.jhu.edu/~sid/pyrosetta/downloads/documentation/PyRosetta_Manual.pdf, particularly Units 2 (Protein Structure in PyRosetta) and 3 (Calculating Energies in PyRosetta), and the Appendix.

In the following, we will provide instructions on how to complete this problem on Athena’s Dialup Service, where PyRosetta has previously been installed as a module for another class.

Log onto Athena’s Dialup Service, either at a workstation on campus or from a Terminal on your personal machine:

ssh <your Kerberos username>@athena.dialup.mit.edu

Once logged on, load the PyRosetta module with the following 2 commands:

cd /afs/athena/course/20/20.320/PyRosetta
source SetPyRosettaEnvironment.sh

If you log onto Athena to complete the problem at a future time, you will have to execute these two commands again, otherwise you will get the following error:

ImportError: No module named rosetta

Now, head to the course's Athena directory, copy the pyRosetta_RefiningStructures.zip folder of files for this problem to your home directory (~), and then unzip it in your home directory with the following commands:

cd /afs/athena/course/7/7.91/sp_2014

cp pyRosetta_RefiningStructures.zip ~

cd ~

unzip pyRosetta_RefiningStructures.zip

cd pyRosetta_RefiningStructures
You should now be able to edit any of these files. Skeleton code is provided in pyRosetta_1EK8.py. The 1EK8.clean.pdb file is a cleaned PDB file of the *E. coli* ribosome recycling factor, while 1EK8.rotated.pdb is the same structure but with some phi and/or psi angles rotated. In this problem, you will refine the 1EK8.rotated.pdb structure using an Energy Minimization approach as well as a Simulated Annealing approach.

**(A – 1 point)** Complete the part_a_energy_minimization() function in pyRosetta_1EK8.py to carry out a simple greedy energy minimization algorithm. Rather than computing the gradient of the full energy potential, you should simply calculate the energy of the structure for each of the residue’s phi and psi angles changed by ±1 degree, and accept the structure with the lowest energy, continuing until the change in energy is less than 1 (see the code for more specific details).

What are the starting and final energies of the structure? How many iterations until convergence? If implemented correctly, a plot of the energy at each iteration (energy_minimization_plot.pdf) will be made; upload it to the course website electronic dropbox or include a printout of it with your write-up. Once finished with this, to cut down on run-time for subsequent parts of the question you may want to comment out the lines between ###### PART (A) ####### and ###### END PART (A) #######.

**(B – 1 point)** Now we’ll refine 1EK8.rotated.pdb through a Simulated Annealing approach. For the initial iterations, we aim for an acceptance criterion of 50% for structures that have twice the starting energy of 1EK8.rotated.pdb. What should we initially set $kT$ (henceforth simply referred to as the temperature since $k$, the Boltzmann constant, remains the same) to be to achieve this acceptance rate?
(C – 2 points) Now let’s implement the core of the Simulating Annealing algorithm: making a number of possible changes to the structure and accepting each according to the Metropolis criterion. To do this, fill in `onesetofMetropolismoves()`, a function that is called by `part_c()`; your implementation of `onesetofMetropolismoves()` should repeatedly call `make_backbone_change_metropolis()`, which you should implement to make one phi or psi angle perturbation drawn from a Normal(0, 20) distribution and accept or reject it according to the Metropolis criterion (see the code for more specific details).

Finally, change `kT_from_part_B = 1` at the bottom of the code to your answer from part (B). If your implementation of the functions called by `part_c()` is correct, `energy_plot_metropolis.pdf` will be produced, showing the energy of the structure at each of the 100*number_of_residues iterations with your kT_from_part_B. Upload it to the course website dropbox or include a printout of it with your writeup.

(D – 2 points) Finally, let’s implement an annealing schedule to make the Simulated Annealing algorithm more realistic:

1. Start at the temperature calculated in part (B).
2. Perform 100*number_of_residues iterations with this kT.
3. Cut kT in half. Repeat step 2.
4. Continue halving until kT is less than 1. You should perform 100*number_of_residues iterations for the first kT value less than 1, and then stop.

Using the temperature calculated in part (B), how many kT halvings should you perform?

In the code, change `number_of_kT_halvings = 1` to this value. Then complete the `part_d()` function to implement the above annealing schedule (see the code for more specific details). If your implementation is correct, `energy_plot_metropolis_part_d.pdf` will be produced, showing the energy of the structure at each of the 100*number_of_residues*(number_of_kT_halvings+1) iterations with the kT at the end of each iteration set labeled on the x-axis. Upload this plot to the course website dropbox or include a printout of it with your writeup. In no more than two sentences, compare/contrast this Simulated Annealing plot with the Energy Minimization plot from part (A). (Note: performing the algorithm with the computed number_of_kT_halvings could take up to an hour, so it’s recommended that you test your code with number_of_kT_halvings=1 first to make sure there are no errors, and then change number_of_kT_halvings to your calculated value to run the Simulated Annealing to completion).
(E – 1 point) Finally, let’s evaluate the structures you’ve produced using two criteria:

1. a simple count of the number of phi and psi angles that differ by more than 1 degree.
2. the root-mean-square deviation (RMSD).

To calculate (1), you should complete the `part_e_num_angles_different()` function. A PyRosetta function can be called to easily calculate (2) (hint: see the Appendix of the linked PyRosetta Documentation).

Then fill out the table below. Which approach (Energy Minimization or Simulated Annealing) appears to have done better here? Can you think of a scenario in which the other approach would do better?

<table>
<thead>
<tr>
<th>Difference between 1EK8.clean.pdb and:</th>
<th>Number of phi and psi angles that differ by &gt;1°</th>
<th>RMSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1EK8.rotated.pdb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Minimization structure from part (A)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulated Annealing structure from part (D)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
P3. Mutual information of protein residues (7 points).

In this problem, you will explore the mutual information of amino acid residues in a Multiple Sequence Alignment (MSA) of the Cys/Met metabolism PLP-dependent enzyme family (http://pfam.sanger.ac.uk/family/PF01053#tabview=tab0). Skeleton code is provided in mutual_info.py in Problem3.zip on course website. The full MSA, which you should use for all of your answers, is cys_met.fasta. However, we have also provided you with a smaller file containing the first ~1000 lines of the MSA (which you can use when developing and testing your code to cut down on the run-time) as cys_met_shortened.fasta.

(A – 2 points) Open mutual_info.py and scan to the bottom (after if __name__ == "__main__":) to get a feeling for what functions are executed and what each function returns. First, we’ll calculate the information content at each position of the alignment. What is maximum information possible at one position (in bits), and what is the formula for information that you will implement?

Maximum information possible:

Formula for information at a position:

Now, complete the part_A_get_information_content() function in mutual_info.py, and plot the information content at each position (see the code for more specific details). The code can be run with the following command:

```
python mutual_info.py cys_met.fasta
```

What is the maximum information content, and what is the first position at which this maximum information is attained?

If you have matplotlib installed (you also can upload your code to Athena, which has matplotlib installed), you can uncomment `plot_info_content_list(info_content_list)`, which will make 2 plots of the information content at each position (one relative to the original MSA positions and a condensed version relative to the well-represented position numbers); otherwise, make a plot of the information content at each position with a tool of your choice. Upload one of the 2 plots (or your own custom one) to the course website electronic dropbox or include a printout of it with your write-up.
(B – 3 points) Now let’s calculate the mutual information between all pairs of well-represented positions in the alignment. Complete the function `get_MI_at_pairs_of_positions()` to get the mutual information at each pair of positions, and plot the mutual information at each pair of positions (if you have matplotlib installed, you can uncomment the `plot_mutual_information_dict(mutual_information_dict)` function provided to make 2 heatmap plots, one relative to the original MSA positions, and a condensed one relative to the “well-represented” position number).

What is the maximum mutual information, and at what pair of positions is this value achieved? Upload the heatmap plot with the well-represented positions to the course website Problem Set 4 electronic dropbox or include a printout of it with your write-up.

(C – 2 points) Now’s lets see if we can make sense of why the positions with the highest mutual information are as such. Complete the function `part_c_get_highest_MI_block_of_10()` to find the 10 consecutive well-represented positions with the highest average mutual information. If your function is implemented successfully, the code will subsequently print out the human sequence (including gaps and intervening non-well-represented positions) corresponding to this block. What is it?

The human entry (CGL_HUMAN/19–395) in the multiple sequence alignment `cys_met.fasta` corresponds to 1QGN (Figures 1-5) in the paper “New methods to measure residues coevolution in proteins” by Gao et al. BMC Bioinformatics 2011, 12:206 (http://www.biomedcentral.com/content/pdf/1471-2105-12-206.pdf). By matching the printed-out, gapped sequence in the MSA human entry to the ungapped human sequence (Isoform 1 of http://www.uniprot.org/uniprot/P32929), determine which positions of the human protein correspond to the highest MI block. Are these positions in any of the figures from the paper? Based on the structure of the enzyme, why would you expect these positions to have high mutual information?

As a reminder, be sure that your final answers are for `cys_met.fasta` (not `cys_met_shortened.fasta`).