1. Let $X$ be a binomial random variable with parameters $n$ and $p$ and let $Y$ be a binomial random variable with parameters $m$ and $p$. Assume that $X$ and $Y$ are independent. Show that $Z = X + Y$ is a binomial random variable with parameters $n + m$ and $p$. (Hint: Use the arguments for the sum of two Poisson random variables in Example 5.4 in Lecture 5 and the relationship $\sum_{i=0}^{n} \left( \begin{array}{l} n \\ i \end{array} \right) \left( \begin{array}{l} m \\ k-i \end{array} \right) = \left( \begin{array}{l} n+m \\ k \end{array} \right)$.)

2. In Example 5.4, we show that if $X$ and $Y$ are two independent Poisson random variables, with parameters $\lambda_1$ and $\lambda_2$, respectively, then $Z = X + Y$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$. Show that the pmf of $X$ given $Z$ is the binomial pmf with $n = z$ and $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$.

   A. First, explain why
   $$Pr(X = x \cap Z = z) = Pr(X = x \cap Y = z - x) = \frac{\lambda_1 e^{-\lambda_1} \lambda_2^{z-x} e^{-\lambda_2}}{(z-x)!}.$$  
   B. Next, to obtain the result compute
   $$Pr(X | Z) = \frac{Pr(X = x, Z = z)}{Pr(Z = z)} = \frac{Pr(X = x, Y = z - x)}{Pr(Z = z)}.$$  

3. Suppose that $X$, $Y$ and $Z$ are independent discrete random variables and that each assumes the values 1, 2 and 3 with probability $\frac{1}{3}$.

   A. Find the pmf of $W = X + Y$.
   B. Find the pmf of $V = Z + W$.

4. The joint probability density (Example 4.5, p.5 of Lecture 5) is difficult to visualize. Therefore, you want to simulate values from this density and make a scatter plot.

   A. Assume that $\lambda = 3$ and use Algorithm 5.1 to simulate in MATLAB 500 draws (i.e. $(X, Y)$ pairs) from $f_{xy}(x, y)$. This will entail first drawing $X$ from the exponential density $f_x(x) = 3e^{-3x}$, using Example 3.2. Then, given $X = x$ draw $Y$ from $f_{xy}(y | x) = 3e^{-3(x+y)}$, again using Example 3.2.
B. Make a histogram plot of the $X$'s. Does this look like what you would expect, i.e., the marginal density of $X$?

C. Make a histogram plot of the $Y$'s. Does this look like what you would expect, i.e., the marginal density of $Y$?

5. The moment generating functions of the random variables in Problem 1 are for $X$, $\phi_x(t) = (pe^t + 1 - p)^n$ and for $Y$, $\phi_y(t) = (pe^t + 1 - p)^m$. Solve Problem 1 using the moment generating functions, that is by finding the moment generating function of $Z = X + Y$.

6. Suppose $X$ has a gamma distribution with parameters $\alpha$ and $\beta$. Using the moment generating function
   
   A. Compute the skewness of $X$.
   
   B. Compute the kurtosis of $X$.

7. The moment generating function of a Gaussian random variable is $\exp(\frac{\sigma^2 t^2}{2})$. Find its fourth moment.