**Problem 1**

Streptomycin is an antibiotic used in combination with other drugs to treat tuberculosis (TB), which is a chronic bacterial infection. Suppose that you are running a fungal fermentation to produce this antibiotic and that the biomass titer at harvest is 20 g/L. To separate the biomass from the streptomycin product, you will harvest the fermentation broth, add filter aid to a concentration of 150 g/L, and proceed to perform the primary separation using a rotary vacuum filter.

In pilot studies using a filter having 1.5 m² area, the pressure drop across the filter is 12 psi and the data shown in Table 1 are collected.

<table>
<thead>
<tr>
<th>Filtration Time (min)</th>
<th>Total Volume Collected (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5.0</td>
</tr>
<tr>
<td>28</td>
<td>20.0</td>
</tr>
<tr>
<td>293</td>
<td>65.0</td>
</tr>
<tr>
<td>375</td>
<td>75.0</td>
</tr>
<tr>
<td>512</td>
<td>85.0</td>
</tr>
</tbody>
</table>

The viscosity of the filtrate is $2.69 \times 10^{-4}$ lb/ft-s and the cake is compressible with a resistance ($R_c$) that can be expressed as follows:

$$R_c = \alpha \rho_o \left( \frac{V}{A} \right)$$

where

- $\alpha$ = specific cake resistance (length/mass)
- $\rho_o$ = mass of cake solids per volume of filtrate
- $V$ = filtrate volume
- $A$ = filter area
a) Derive an equation that relates area (A), time (t), and volume (V) to area and volume in the following functional form (f means function of):

\[
\frac{At}{V} = f\left(\frac{V}{A}\right)
\]

b) Use your answer from part a to determine the value of \(\alpha\) (in units of ft/lb) and the filter resistance \(R_m\) (in units of ft\(^{-1}\)).

c) If you are to scale up this operation to filter 10 m\(^3\) of fermentation broth in 10 hours and the maximum filter area per rotary vacuum filter machine is 80 m\(^2\), how many machines must be used in parallel?

**Problem 2**

A tubular membrane is being used for ultrafiltration of cheese whey. In the absence of protein, the membrane has a filtration of 100 L/m\(^2\)-hr when the transmembrane pressure is 0.4 atm. The whey proteins have an average diffusivity of 4 x 10\(^{-7}\) cm\(^2\)/s, and the osmotic pressure in atmospheres is given by Jonsson’s equation:\(^1\):

\[
\pi = (4.4 \times 10^{-3})C - (1.7 \times 10^{-6})C^2 + (7.9 \times 10^{-8})C^3
\]

where C is the protein concentration in grams per liter. Based on the channel geometry, flow conditions, and whey diffusivity, the protein mass transfer coefficient, k, is 7.8 x 10\(^{-4}\) cm/s.

a) Calculate the phenomenological hydraulic permeability, \(L_P\), for the transport of water across the membrane.

b) Calculate the effect of \(\Delta P\) on the membrane flux if the protein concentration is 10 g/L. Please show the effect by creating a plot of flux (L/m\(^2\)-hr) vs. \(\Delta P\) (atm) for fluxes ranging from 10 to 104 L/m\(^2\)-hr. You may assume that protein is completely rejected from the membrane. (*Hint: this problem should be done using Excel*)

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Problem 3

In Problem 2 you determined the effect of $\Delta P$ on ultrafiltration membrane flux for cheese whey being filtered at a concentration of 10 g/L. In this problem your objective is to analyze the effect of bulk protein concentration on membrane flux.

If the calculations performed in Problem 2 were done for several protein concentrations, the following type of plot could be generated:

[Image removed due to copyright reasons.]

Please see:

As shown in the figure above, at a specific transmembrane pressure a different flux is obtained for each bulk protein concentration. In the region in which flux is independent of transmembrane pressure, a gel layer exists that limits the maximum flux that can be achieved.

The data in Table 2 were collected from the ultrafiltration of a protein at several different concentrations when the flux was pressure independent.

Table 2: Ultrafiltration Flux as a Function of Bulk Protein Concentration

<table>
<thead>
<tr>
<th>Bulk Protein Concentration (g/L)</th>
<th>Filtration Flux (L/m²-hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>103.8</td>
</tr>
<tr>
<td>100</td>
<td>39.5</td>
</tr>
<tr>
<td>200</td>
<td>20.0</td>
</tr>
<tr>
<td>300</td>
<td>8.8</td>
</tr>
</tbody>
</table>

a) Please use the data in Table 2 to calculate the gel layer concentration, \( C_g \).

b) What is the maximum filtration flux that can be achieved by increasing pressure for whey protein being filtered at a concentration of 20 g/L? Please use the data for the filtration of cheese whey in Problem 2 to help answer this question.

**Problem 4**

A large-volume application of membranes is ultrafiltration in the dairy industry in which milk is preconcentrated for the manufacture of soft cheeses: mozzarella, camembert, brie, fetta, and cottage cheeses. You are consulting for a cheese manufacturer that is performing ultrafiltration of milk at 50 °C. The data available for the membrane used in the process are as follows:

\[
\begin{align*}
\text{Hydraulic Channel Diameter (d_h)} &= 0.11 \text{ cm} \\
\text{Channel Length (L)} &= 63.5 \text{ cm}
\end{align*}
\]

The physical properties of milk are as follows:

\[
\begin{align*}
\text{Liquid Density} &= 1.03 \text{ g/cm}^3 \\
\text{Liquid Viscosity} &= 0.008 \text{ g/cm-sec} \\
\text{Protein Diffusivity (D)} &= 7.0 \times 10^{-7} \text{ cm}^2/\text{second} \\
\text{Bulk Protein Conc. (C_b)} &= 3.1\% \\
\text{Gel Protein Conc. (C_g)} &= 22\%
\end{align*}
\]

It has been found that it takes 24.7 minutes to process 1,000 L of milk using 100 m² of membrane when the fluid velocity is maintained at 100 cm/s and the filtration flux is pressure independent. If rescheduling of the process requires that 1,000 L of milk be processed in 60 minutes by only changing the channel fluid velocity, what must the new velocity be? (Hint: Laminar flow in the filtration apparatus occurs for Reynolds numbers below ~ 2,300).