MEMBRANE TRANSPORT

Transport

Membrane materials and structure create barrier and 
semipermeable membrane barrier separating two fluid 
mixtures in which chemical potentials of one or more 
components differs

$\Delta$ Chemical potentials creates driving force

**Types of Transport**

- **Passive:** Down chemical potential gradient
- **Facilitated:** Carrier mediated transport in which one or more species binds to diffusible carrier
- **Active:** Up chemical potential gradient, coupled to metabolic chemical reaction
Chemical Potential of Species $i$ ($\mu_i$)

$$u_i = u_i^\circ + \overline{S}_i T + \overline{V}_i P + RT \ln a_i + Z_i F \Psi + ...$$

$u_i^\circ$ = chemical potential of standard state

$\overline{S}_i$ = partial molal entropy

$\overline{V}_i$ = partial molal volume

$R$ = ideal gas constant

$Z_i$ = valence

$F$ = Faraday's constant

$P$ = Pressure

$a_i = \gamma_i C_i$, chemical activity

$\gamma_i$ = activity coefficient

$C_i$ = concentration

$\Psi$ = electrical field potential

Driving forces: $T$, $P$, $C$, $\psi$, ...
Two or more may act simultaneously
# Categorization of Membrane Separation Processes

<table>
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<tr>
<th>Name</th>
<th>Upstream Mixture</th>
<th>Downstream Mixture</th>
<th>Driving Force</th>
<th>Permeant</th>
<th>Rejected Species</th>
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<tr>
<td>Gas Permeation</td>
<td>Gas</td>
<td>Gas</td>
<td>Concentration or Partial Pressure</td>
<td>Gas</td>
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<tr>
<td>Pervaporation</td>
<td>Liquid Solution</td>
<td>Gas</td>
<td>Concentration or Partial Pressure</td>
<td>Gas</td>
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<td>Dialysis</td>
<td>Liquid Solution</td>
<td>Liquid Solution</td>
<td>Concentration</td>
<td>Microsolute</td>
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<tr>
<td>Electrodialysis</td>
<td>Liquid Solution</td>
<td>Liquid Solution</td>
<td>Electrical Potential</td>
<td>Ions</td>
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<tr>
<td>Filtration</td>
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<tr>
<td>Reverse Osmosis</td>
<td>Liquid Solution</td>
<td>Liquid Solution</td>
<td>Pressure and Concentration</td>
<td>Solvent</td>
<td>Microsolute</td>
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<tr>
<td>(Hyperfiltration)</td>
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<td>Ultrafiltration</td>
<td>Liquid Solution</td>
<td>Liquid Solution</td>
<td>Pressure and Concentration</td>
<td>Microsolute</td>
<td>Macrosolute</td>
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<tr>
<td>Microfiltration</td>
<td>Liquid Suspension</td>
<td>Liquid Solution</td>
<td>Pressure and Concentration</td>
<td>Macrosolute</td>
<td>Colloidal Particles</td>
</tr>
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<td>Particle Filtration</td>
<td>Liquid Suspension</td>
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<td>Pressure and Concentration</td>
<td>Colloidal Particles</td>
<td>Macroscopic Particles</td>
</tr>
</tbody>
</table>
STEADY-STATE
CONVECTIVE TRANSPORT

Hydrostatic Pressure Driving Force

Volume Flux \( J_V = \frac{k\Delta P}{\mu L} \)

Darcy's Law \( \left( \frac{k}{\mu L} \right)\Delta P = L_p \Delta P \)

- \( k \) = hydraulic permeability
- \( \mu \) = viscosity
- \( P \) = hydrostatic pressure
- \( J_V = L_p \Delta P \)
- \( L_p \) = phenomenological hydraulic permeability = \( \frac{k}{\mu L} \)
### STEADY-STATE DIFFUSIVE TRANSPORT

**Partitioning** of solute between solution and membrane

\[ K_p = \frac{C_m}{C_s} \]

**Mass Flux**

\[ J_s = -D \frac{dC}{dx} \]

- \( D \) = effective diffusion coefficient in membrane
- \( C \) = concentration
- \( x \) = distance

Integration across membrane:

\[ J_s = D \frac{C_{m1} - C_{m2}}{L} = DK_p \frac{C_{s1} - C_{s2}}{L} \]

\( K_p \) = membrane partition coefficient = \( \frac{C_m}{C_s} \)

\[ J_s = P_m \Delta C_s \]

\( P_m \) = membrane permeability = \( \frac{DK_p}{L} \)

Thus,

\[ J_s = \frac{DK_p}{L} (C_{s1} - C_{s2}) = P_m (\Delta C_s) \]
COMBINED DIFFUSIVE CONVECTIVE TRANSPORT
THE LINEAR CASE

Basis: Thermodynamics of Irreversible Processes

Key Assumption: Small departure from equilibrium

\[
\frac{d \mu_i}{dx} = \frac{\Delta i \mu}{\Delta X}
\]

\[
J_V = \bar{L}_p (\Delta P - \sigma \Delta \pi)
\]

The greater the rejection the greater the impact of $\Delta \pi$

\[
J_s = P_m \Delta C_s + (1 - \sigma) \bar{C} J_V
\]

This describes solute passes this membrane by convective flow

\[
\bar{C} = \text{average solution concentration}
\]

\[
\sigma = \text{Staverman reflection coefficient} = \frac{\text{actual} \Delta \pi}{\Delta \mu \text{ with } \sigma = 1}
\]

Limiting Cases:
\[
\sigma = 1 \text{ Membrane completely retains solute}
\]
\[
\sigma = 0 \text{ membrane freely permeable to solute}
\]
REGIMES OF CROSS-FLOW FILTRATION

\[ J_U = J_F = L_P \Delta P \quad L_P = \frac{K}{\mu L} \]

MEMBRANE HYDRAULIC PERM. IS CONSTANT

FORMATION OF CP LAYER WITH VARIABLE "K"

Pressure - Dependent  \quad \text{Pressure - Independent}

Membrane - Limited  \quad \text{Boundary Layer - Limited}

PRESSURE DIFFERENCE, \( \Delta P \)
MASS TRANSFER MODEL FOR MEMBRANE FILTRATION

At Steady State

\[ J C_b = D \frac{dC}{dx} \]

Integration across the boundary layer \( \delta \)

\[ J = \frac{D}{\delta} \ln \frac{C_g}{C_b} = k \ln \frac{C_g}{C_b} \]

\[ J = f(k) = f(V_s, \mu, P, \text{system geometry}) \]

Mass Transfer coefficient

\[ K = f(Re, S_c) \]

\[ Re = \frac{\nu d P}{\mu} \]

\[ S_c = \frac{\mu}{P D} \]
CELL HARVESTING/ E. coli
PROSTAK-1

- + - Flux
- - Δ -- Temperature

FLUX (L/m²/h)

Temperature (°C)

TIME (min)
CELL HARVESTING/ E. coli

HF-LAB-5/ROMICON-1

- + - Flux
- Δ - Temperature

FLUX (L/m²/h)

Temperature (°C)

TIME (min)
Cell harvesting of E. coli
Membrane filtration

- Pacesetter
- Prostak
- Millipore
- Romicon

Temperature increase
25.5-28.5 °C