Kinetics of Elementary Reactions: Radioactive Decay

See lecture 31 for an introduction to kinetics and lecture 32 for the kinetics of radioactive decay.

Radioactive Decay. The decay of a nucleus is independent of the number of surrounding nuclei that have decayed. We can apply first order integrated rate laws:

\[ [A] = [A]_0 e^{kt} \quad \text{and} \quad t_{1/2} = \frac{0.6931}{k} \]

However, instead of concentration, the first order integrated rate law is expressed in terms of \( N \) (number of nuclei):

\[ N = N_0 e^{kt} \quad \text{\( k \) = decay constant} \]
\[ \quad \text{\( t \) = time} \]
\[ \quad \text{\( N_0 \) = number of nuclei originally present} \]

Nuclear kinetics – monitor rate of occurrence of decay events with a Geiger counter (radiation detector). Decay rate is also called Activity (\( A \))

\[ N = N_0 e^{-kt} \quad \text{can be expressed as} \quad A = A_0 e^{-kt} \quad A = \text{Activity} \quad A_0 = \text{initial Activity} \]

Example from pg 3 of Lecture 32 notes: Medical Applications of Radioactive Decay.

Technetium(Tc)-99 is the most widely used radioactive nuclide in medicine. It is used for diagnostic organ imaging and bone scans, with over 7 million uses annually in the US.

One of the patent holders for the technetium-based imaging agent cardiolite\textsuperscript{TM} is MIT Professor of Chemistry Alan Davison.

Cardiolite\textsuperscript{TM} is a coordination complex, and Prof. Davison figured out which ligands to use (CN\textsuperscript{-}) to obtain the desired properties of solubility and stability to be applied to medical imaging. Cardiolite has saved many lives in diagnosing coronary artery disease.

In a Cardiolite stress test, the molecule is administered by IV and travels through the blood into the heart. Since the drug cannot access areas of the heart with insufficient blood supply, a subsequent scan reveals any blocked arteries.
Recitation or homework example:

*Calculate the total activity (in disintegrations per second) caused by the decay of 0.5 microgram of $^{99m}$Tc (an excited nuclear state of $^{99}$Tc), which has a half-life of 6.0 hours.*

To calculate the activity of a sample of 1.0 mg of $^{99m}$Tc, we can use the following equation: $A=kN$. We need to first determine the decay constant, $k$ and the number of nuclei.

To calculate the number of nuclei:

$\# \text{ of nuclei} = \left( 0.5 \times 10^{-6} \text{ g} \right) \left( \frac{\text{mol}}{99.00 \text{ g}} \right) \left( \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \right)$

$= 3.0414 \times 10^{15}$

To calculate the decay constant:

$t_\frac{1}{2} = 6.0 \text{ hrs} \left( \frac{60 \text{ min}}{\text{hr}} \right) \left( \frac{60 \text{ sec}}{\text{min}} \right)$

$= 2.16 \times 10^{4} \text{ s}$

$k = \frac{\ln 2}{t_\frac{1}{2}}$

$= \frac{0.6931}{2.16 \times 10^{4} \text{ s}}$

$= 3.2088 \times 10^{-5} \text{ s}^{-1}$

We can now substitute those values into the equation $A=kN$;

$A = kN$

$= \left( 3.2088 \times 10^{-5} \text{ s}^{-1} \right) \left( 3.0414 \times 10^{15} \text{ nuclei} \right)$

$= 9.759 \times 10^{10}$

$= 1 \times 10^{11} \text{ disintegrations per second}$

A sample of 0.5 $\mu$g of $^{99m}$Tc has the activity of $1 \times 10^{11}$ disintegrations $s^{-1}$.