Problem Set #1

1. (a) Make the necessary conversions in order to fill in the table:

<table>
<thead>
<tr>
<th>Wavelength (Å)</th>
<th>420</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavenumber (cm⁻¹)</td>
<td>100</td>
</tr>
<tr>
<td>Energy (J)</td>
<td></td>
</tr>
<tr>
<td>Energy (kJ/mole)</td>
<td>490</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>8.21 × 10¹³</td>
</tr>
</tbody>
</table>

(b) Name the spectral region associated with each of the last four columns of the table.

2. A 100-W tungsten filament lamp operates at 2000 K. Assuming that the filament emits like a black-body, what is the total power emitted between 6000 Å and 6001 Å? How many photons per second are emitted in this wavelength interval?

3. (a) What is the magnitude of the electric field for the beam of a 1mW helium-neon laser with a diameter of 1 mm?

(b) How many photons per second are emitted at 6328 Å?

(c) If the laser linewidth is 1 kHz, what temperature would a blackbody have to be to emit the same number of photons from an equal area over the same frequency interval as the laser?

4. The lifetime of the 3²P₃/₂ → 3²S₁/₂ transition of the Na atom at 5890 Å is measured to be 16 ns.

(a) What are the Einstein A and B coefficients for the transition?

(b) What is the transition dipole moment in Debye?

(c) What is the peak absorption cross section for the transition in Å², assuming that the linewidth is determined by lifetime broadening?

5. (a) For Na atoms in a flame at 2000 K and 760 Torr pressure, calculate the peak absorption cross section (at line center) for the 3²P₃/₂ → 3²S₁/₂ transition at 5890 Å. Use 30 MHz/Torr as the pressure-broadening coefficient and the data in Problem 4.

(b) If the path length in the flame is 10 cm, what concentration of Na atoms will produce an absorption (I/I₀) of 1/e at line center?

(c) Is the transition primarily Doppler or pressure broadened?

(d) Convert the peak absorption cross section in cm² to the peak molar absorption coefficient ε.

6. For Ar atoms at room temperature (20° C) and 1 Torr pressure, estimate a collision frequency for an atom from the van der Waals radius of 1.5 Å. What is the corresponding pressure-broadening coefficient in MHz/Torr?
7. Solve the following set of linear equations using matrix methods

\[\begin{align*}
4x - 3y + z &= 11 \\
2x + y - 4z &= -1 \\
x + 2y - 2z &= 1.
\end{align*}\]

8. (a) Find the eigenvalues and normalized eigenvectors of the matrix

\[A = \begin{pmatrix} 2 & 4 - i \\ 4 + i & -14 \end{pmatrix}.\]

(b) Construct the matrix \(X\) that diagonalizes \(A\) and verify that it works.

9. Given the matrices \(A\) and \(B\) as

\[A = \begin{pmatrix} -\frac{1}{3} & \sqrt{\frac{2}{3}} & \frac{\sqrt{3}}{3} \\ \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{7}}{3} & \frac{1}{\sqrt{3}} & -\frac{2}{3} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{5}{3} & \frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{3}} & \frac{11}{6} \end{pmatrix}.\]

Show that \(A\) and \(B\) commute. Find their eigenvalues and eigenvectors, and obtain a unitary transformation matrix \(U\) that diagonalizes both \(A\) and \(B\).

10. Obtain eigenvalues and eigenvectors of the matrix

\[H = \begin{pmatrix} 1 & 2\alpha & 0 \\ 2\alpha & 2 + \alpha & 3\alpha \\ 0 & 3\alpha & 3 + 2\alpha \end{pmatrix}\]

to second order in the small parameter \(\alpha\).

11. A particle of mass \(m\) is confined to an infinite potential box with potential

\[V(x) = \begin{cases} \infty, & x < 0, x > L, \\
k \left(1 - \frac{x}{L}\right), & 0 \leq x \leq L. \end{cases}\]

Calculate the ground and fourth excited-state energies of the particle in this box using first-order perturbation theory. Obtain the ground and fourth excited-state wavefunctions to first order, and sketch their appearance. How do they differ from the corresponding unperturbed wavefunctions?