1.020 Ecology II: Engineering for Sustainability

Lectures 08_19 Derived Supply, Equilibrium, Groundwater Pumping

Motivation/Objective
Extend the derived demand model of Lectures 08_16 & 08_17 to include the cost of supplying water from a single groundwater well. Combine derived supply and demand to obtain an equilibrium solution.

Approach
1. Derive an expression for the marginal cost of supplying water from a single groundwater well. Identify factors affecting pumping cost.
2. Add the supply curve calculation to the MATLAB program of Lectures 08_16 & 08_17
3. Identify the equilibrium solution (intersection of supply and demand curves)
4. Consider how problem inputs (aquifer transmissivity, energy cost, etc.) affect solution.

Concepts and Definitions Needed:
Pumping cost at supply well depends on flow rate and depth to water:
\[ F_{\text{cost}} = p \dot{E} \Delta t = p \rho g \Delta t (z_g - h) Q \] ($\text{season}^{-1}$) \quad \rho = \text{energy price ($\text{joule}^{-1}$)}
\[ \dot{E} = \text{power required to pump water (watts)}, \quad \Delta t = \text{length of irrigation season (sec season}^{-1}) \]
\[ Q_i = \text{Pumping rate at } i \text{ (m}^3\text{ sec}^{-1} \text{)}, \quad z_g = \text{ground surface elev. (m)} \quad h = \text{groundwater elev. (m)} \]

Pumping at the well affects \( h \). Derive \( h(Q) \) using rate form of mass balance eq in radial coords:
Mass rate pumped from well = mass rate flowing through soil toward well
Mass balance:
\[ \rho Q = \rho 2\pi r q \] (kg sec\(^{-1}\)) \quad q = T \frac{dh(r)}{dr} = \text{radial flow per unit width (m}^3\text{ m}^{-1}\text{sec}^{-1})
\[ T = \text{Soil transmissivity (m}^2\text{ sec}^{-1}) \]
Substitute \( q \) in mass balance to get differential eq. for \( h \):
\[ \frac{dh(Q)}{dr} = \frac{Q}{2\pi r T} \quad \text{Solve for } h(Q): \quad h(Q) = h_0 + \frac{Q}{2\pi T} \ln \left( \frac{r_w}{r_0} \right) \quad r_0 >> r_w \]
\( r_w = \text{well radius (m)} \quad r_0 = \text{distance where } h(Q) = h_0 = \text{water level at well when } Q = 0 \)
Marginal cost of supplying water:
\[ \mu(Q) = \frac{\partial F_{\text{cost}}}{\partial Q} = \frac{\partial}{\partial Q} \left[ p \rho g \Delta t [z_g - h(Q)] Q \right] = p \rho g \Delta t \left[ z_g - h_0 - \frac{Q}{\pi T} \ln \left( \frac{r_w}{r_0} \right) \right] \quad ($m^{-3}$)
Include this expression in crop allocation optimization code from Lectures 08_16 & 08_17.
Plot of \( \mu(Q) \) vs \( Q \) gives derived supply (a curve).
Intersection of supply \( \mu(Q) \) and demand \( \lambda(Q) \) defines equilibrium solution \( [Q_{eq}, \lambda_{eq}] \).

Crop Allocation Results with Groundwater Supply Cost Included
Supply curve is linear in \( Q \). Note dependence of equilibrium solution on problem inputs (price, transmissivity, unpumped depths to groundwater, etc.)