Introduction: See course information sheet

Motivation/Objective:
Develop a model to compute time-varying phosphorous concentrations in a stormwater detention pond. Examine effect of pond size.

Approach:
1. Define system, control volume (CV), system properties, identify unknown (dissolved pond phosphorous concentration $C_t$ at time $t$)
2. Write mass balance equation (incremental, over a specified time interval)
3. Relate boundary and gain/loss terms in mass balance equation to unknown $C_t$.
4. Specify all inputs, solve mass balance equation for unknown (MATLAB)
5. Use model to evaluate how different pond characteristics affect phosphorous concentration.

Concepts and Definitions:
Definitions: system, surroundings, control volume
Distinguish isolated, closed, open systems
Thermodynamic systems are described by bulk system properties, processes
Distinguish mass inflows/outflows across CV boundary from internal gains/losses.
Mass Conservation:
- Incremental form: $\Delta M_{cv} = \Delta M_{in} - \Delta M_{out} + \Delta M_{gain} - \Delta M_{loss}$ over $[t, t+1]$
- Instantaneous (or rate) form: $\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out} + \dot{m}_{gain} - \dot{m}_{loss}$
- Steady-state: $\Delta M_{cv} = \frac{dM_{cv}}{dt} = 0$
Distinguish conservative vs non-conservative system properties.
Relationship between mass and concentration: $M_{cv} = V_{cv} C_{cv}$

For Everglades example (open system):
$\Delta M_{cv} = V_{ci+1} - V_{ci}$, $\Delta M_{in} = q_{in} C_{in} \Delta t$, $\Delta M_{out} = q C_i \Delta t$, $\Delta M_{loss} = \alpha C \Delta t$, $\Delta M_{gain} = 0$
$q$ = flow rate (m$^3$ hr$^{-1}$) (inflow = outflow), $C_{in}$ = phosphorous concentration in inflow

Final Mass Balance Equation (expressed in terms of the unknown concentration $C_t$):
$C_{i+1} = C_i \left( \frac{\Delta t}{\tau} + \left[ 1 - \frac{\Delta t}{\tau} - \alpha \tau \right] \right) C_i$, $\tau = \frac{V}{q}$ = residence time, $\Delta t$ = interval (hrs) between $t$ and $t+1$

Model Results
Note impact of pond volume and plant uptake rate on magnitude and variability of pond phosphorous concentrations.