3.1 Erdős Rényi graphs

In this homework we will explore numerically some of the properties of Erdős Rényi (ER) graphs. For this we will create ER graphs with \( n = 5000 \) nodes, and increasing probability \( p \).

(a) \([20 \text{ points}]\) Create a number of ER graphs with increasing probability \( p \). Your probabilities should cover the range \( p = [10^{-5}, 10^{-2}] \) with ‘logspace’ (\texttt{np.logspace}). Plot the size of the largest connected component relative to the number of nodes \( n \) (i.e., if the giant component consists of the whole graph its size is 1), as a function of \( p \). For each \( p \) you should create 20 graphs and plot the mean value plus / minus the standard deviation of the giant component size.

(b) \([10 \text{ points}]\) What do you observe for \( p \approx 1/n \)? What happens for \( p \approx \log(n)/n \)? Provide a brief description of these phenomena in terms of what they imply for the graphs generated with these parameters (5 sentences).

(c) \([20 \text{ points}]\) For the same ER graphs you have generated in part (a), plot the number of triangles of the graph as a function of \( c = p(n-1) \). For each \( c \), use the 20 graphs generated in part (a) and only plot the mean of the number of triangles. Can you suggest a formula for the expected number of triangles in an Erdős Rényi graph with mean degree \( c \) for large \( n \)? Justify your answer!