Why Sandcastles Fall? We want to study the stress fields in a dry and a humid sandpile, idealized as an inclined semi-infinite half-space oriented at an angle $\alpha$ to the horizontal (see figure below). We choose an $x-z$ coordinate system, in which $z$ gives the distance from the surface of the pile ($z > 0$ down) and $x$ gives the distance parallel to the surface (infinite extension in the $y-$ direction). The sandpile is subjected to its deadweight (volume mass density $\rho$, and $g$ the earth acceleration vector), and static evolutions are assumed.

Problem Set: Mohr-Coulomb’s problem — idealized problem of a sandpile.
1. **Dry Sandpile – The Mohr-Coulomb result**: We restrict ourselves to solutions which are functions of \( z \) alone, i.e.,
\[
\sigma = \sigma(z)
\]
Furthermore, the sand behavior is assumed isotropic.

(a) Determine precisely the conditions which stress field \( \sigma \) needs to satisfy in order to be statically admissible. Determine the non-zero stress components of \( \sigma \), and give a precise of the stress components of which the value is not given by static equilibrium (S.A.-stress conditions).

(b) For a given distance \( z > 0 \) from the surface, represent the previously determined stress state in the Mohr Stress plane. In this plane, indicate the angle \( \alpha \).

(c) We want to provide the critical angle \( \alpha \leq \max \alpha \), by considering that the material in the sandpile obeys to the (dry sand) Mohr-Coulomb criterion:
\[
|\tau| + \sigma \tan \varphi \leq 0
\]
where \( \tau \) is the tangential stress across some plane interior to the sandpile, \( \sigma \) is the normal stress across the same plane, and \( \tan \varphi \) is the internal friction angle. Show the criterion in the Mohr space, and determine the critical value of \( \alpha \) at which the material reaches the Mohr-Coulomb criterion.

2. **Humid Sandpile**: Consider now a sandpile in which a normal adhesive stress \( s_A \) is exerted across every plane, in addition to whatever other stresses may exist due to body forces. This adhesive stress introduces a normal force between pairs of contiguous particles which allows the sandpile to support a finite shear stress (i.e. \( \tau \)), even in the limit of zero applied compressive stresses (i.e. \( \sigma = 0 \)). The maximum shear stress, in this case, is \( \max |\tau| = s_A \tan \varphi \).

(a) Propose a modified Mohr-Coulomb criterion, which for \( s_A = 0 \) gives the dry sand Mohr-Coulomb criterion.

(b) In comparison with the dry sand criterion, how does the Mohr plane representation change in the case of a humid sandpile. Determine the critical angle at which the material reaches the humid sand failure criterion. In comparison with the dry sandpile, does \( \max \alpha \) increase or decrease? Conclude by suggesting how sandcastles fall.