1.033/1.57

Mechanics of Material Systems
(Mechanics and Durability of Solids I)

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Lecture: MWF1 // Recitation: F3:00-4:30
Part II: Momentum Balance, Stresses and Stress States

4. Stress States / Failure Criteria
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Part I. Deformation and Strain
  1 Description of Finite Deformation
  2 Infinitesimal Deformation

Part II. Momentum Balance and Stresses
  3 Momentum Balance
  4 Stress States / Failure Criterion

Part III. Elasticity and Elasticity Bounds
  5 Thermoelasticity,
  6 Variational Methods

Part IV. Plasticity and Yield Design
  7 1D-Plasticity – An Energy Approach
  8 Plasticity Models
  9 Limit Analysis and Yield Design
Stress Vector and Stress Components

Stress components on a material surface oriented by unit normal \( \mathbf{n} \)

Stress components on a material surface oriented in the principal stress direction \( \mathbf{n} = \mathbf{u}_I \)

\[
T(\mathbf{n}) = \sigma_I \mathbf{n}
\]

\[
t = \mathbf{u}_{II}
\]
Stress Vector in the Principal Stress Space

(Illustration of Stress Invariants)

\[ T(-u_1) \]
\[ T(-u_2) \]
\[ T(-u_3) \]
\[ T(n) = \sigma_m n + \tau_{oct} t \]

\( n \) = Orientation of hydrostatic axis

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Stress Vector in the Mohr Plane

\[ T(n) = \sigma(n)n + \tau(n,t)t \]

\[ \Theta \]

\[ \sigma(n)n \]

\[ \tau(n,t) \]

\[ u_I \]

\[ u_{II} \]
Mohr Circles and *The* Mohr Circle

The Mohr Circle

\[ \sigma_{III} \]

\[ \sigma_{II} \]

\[ \sigma_I \]
Selected Stress States: Hydrostatic Pressure

\[ \sigma = -p 1 \]
Selected Stress States: Uniaxial Tension

\[ \sigma = \sigma_I e_z \otimes e_z \]

Material Plane

Mohr Stress Plane

\( n = e_z \)

\( \sigma_I \)

\( \sigma_{II} = \sigma_{III} \)

OT\( (e_z) \)
Selected Stress States: Pure Shear

\[ \sigma = \tau (e_x \otimes e_y + e_y \otimes e_x) \]

\( t = e_y, \quad n = e_x, \quad \theta = -\pi/4 \)

Material Plane

Mohr Stress Plane

\(-2\theta = \pi/2\)
Selected Stress States: Plane Stress

\[
T(n=e_z) = \sigma \cdot e_z = 0
\]

Material Plane

Mohr Stress Plane

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Tension Cut-Off

\[ \sigma = f'_t \]

Shear

Direct Tension

\( -p \)

\( -\tau_{\text{max}} \)
### Tresca Criterion

\[ \theta(u_{III}, n) = +\pi/4 \]

\[ \theta(u_I, n) = -\pi/4 \]

\[ \sigma_0 / 2 \]

\[ -\sigma_0 / 2 \]
Tresca Criterion: Application

\[ \sigma_{II} = \sigma_{III} = -p \]

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\[ \theta = \varphi/2 - \pi/4 < 0 \]

MOHR-COULOMB
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Mohr-Coulomb

Direct Tension
Shear
Uniaxial Compression

\[ +c \]
\[ -c \]
Training Set: Excavation Set

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Max. Excavation Depth of Tresca Material

\[ \sigma_0/2 \]

\[ \tau \]

\[ \sigma \]

\[ -\sigma_0/2 \]

\[ -\rho \gamma x \]

\[ -\rho \gamma (x-H) \]

\[ -\rho \gamma H \]

Zone 1

Zone 2

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Homework Set #2

Part I: Triaxial Test

Part II: Circular Foundation