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Mechanics of Material Systems
(Mechanics and Durability of Solids I)

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Lecture: MWF1 // Recitation: F 3:00-4:30
Part IV: Plasticity and Yield Design

8. Plasticity Models
Part I. Deformation and Strain
  1 Description of Finite Deformation
  2 Infinitesimal Deformation

Part II. Momentum Balance and Stresses
  3 Momentum Balance
  4 Stress States / Failure Criterion

Part III. Elasticity and Elasticity Bounds
  5 Thermoelasticity,
  6 Variational Methods

Part IV. Plasticity and Yield Design
  7 1D-Plasticity – An Energy Approach
  8 Plasticity Models
  9 Limit Analysis and Yield Design

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1D →

Stress $\sigma$, Strain $\varepsilon$
Plastic Strain $\varepsilon^p$
Hardening Variable $\chi$
Hardening Force $\zeta$

$f = |\sigma + \zeta| - k \leq 0$

$\varphi dt = \sigma d\varepsilon - d\Psi \geq 0$

$\varepsilon[L] \quad \sigma$

$E$

$1[L]$

$k$

$H$

$\varepsilon^p$

$\chi$

$\zeta(\chi)$

etc
1D → 3D Extension

Stress $\sigma$, Strain $\varepsilon$

Plastic Strain $\varepsilon^p$

Hardening Variable $\chi$

Hardening Force $\zeta$

\[ f = |\sigma + \zeta| - k \leq 0 \]
\[ \varphi dt = \sigma d\varepsilon - d\Psi \geq 0 \]

etc etc

Stress Tensor $\sigma$, Strain $\varepsilon$

Plastic Strain Tensor $\varepsilon^p$

Hardening Variables $\chi$, $\chi$

Hardening Forces $\zeta$, $\zeta$

\[ f = |s + \zeta| - k \leq 0 \]
\[ \varphi dt = \sigma : d\varepsilon - d\Psi \geq 0 \]

etc etc
Convexity of Elasticity Domain

\[ \frac{\partial f}{\partial \sigma} \]

(a)

(b)

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Example: Crystal Structure of Steel

Impurity = defects in the crystal structure / network

Overcome an Obstacle “when?”

Slippage planes = Direction of permanent deformation “how?”

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Sliding in a Monocrystal (“Kinematics”)
\[ T(n) = \sigma_m n + \tau_{oct} t \]

\[ T(-u_1) \]

\[ T(-u_2) \]

\[ T(-u_3) \]

\( n \) = Orientation of hydrostatic axis

Stress Vector on Deviator Plane
Von-Mises Plasticity: Yield Criterion

Principal Stress Space

Deviator Plane

$\sigma_{I}$ $\sigma_{II}$ $\sigma_{III}$

$D_{E}$

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Von-Mises Plasticity: “Kinematics”

\[ \tau = 2 \varepsilon_{nt} \]

Shear Test

During Plastic Loading

After Unloading

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Drucker-Prager Plasticity: Yield Criterion

\[ \sigma_{II}, \sigma_{I}, \sigma_{III} \]

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Drucker-Prager Plasticity: “Kinematics”

During Plastic Loading

\[ \theta/2 = \varepsilon_{nt} \]

After Unloading

\[ \theta^p/2 = \varepsilon_{nt} \]

\[ \Omega^+ > \Omega^- \]

Plastic Dilatation
Drucker-Prager Plasticity: Thermodynamic Restrictions

Associated Plasticity

Non-Associated Plasticity

Thermodynamically Admissible
Plastic Hardening Models

Isotropic Hardening

\[ f(\sigma, \zeta) \leq 0 \]

\[ |\sigma| = z |\sigma^0| \]

Kinematical Hardening

\[ f(\sigma, \zeta) \leq 0 \]

\[ |\sigma - z| = |\sigma^0| \]
Cam-Clay Model: Yield Criterion

\[ \sqrt{3J_2}/m \]
Cam-Clay Model: “Kinematics”

\[ \sqrt{3J_2/m} \]

\( z = 1.5 \)

\( z = 1 \)

\( z = 0.5 \)

\( \text{tr} d\varepsilon^p < 0 \)

\( \text{tr} d\varepsilon^p = 0 \)

\( \text{tr} d\varepsilon^p > 0 \)

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