Deterministic Planning II & Probabilistic Planning I

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Announcements

- Posted
  - Primavera tutorials
  - Complicated scheduling case
  - Problem set 4 (Scheduling; due Monday April 5)
- Wednesday guest lecture on behavioral managerial issues
Recall: AON (PDM) Scheduling

- Activities shown on nodes
- $O(n)$ Forward/backward pass to determine ES/EF/LS/LF
- Multiple types of relationships
  - FS, SS, FF, SF
- No dummy arrows required
## Example Applications

<table>
<thead>
<tr>
<th>TASK NO.</th>
<th>TASK NAME</th>
<th>DURATION (in days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Place and Secure Trusses</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Install Roof Deck</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Apply Vapor Barrier</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Apply Roof Cladding</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Construct Roof Overhang</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Install Soffits</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Apply Flashing</td>
<td>6</td>
</tr>
</tbody>
</table>
Example Of CPM Algorithm

The AON Network for the Roof Construction

Legend

Early Start Time
Early Finish Time
Late Start Time
Late Finish Time
Task Duration (Days)
Task No.

START
DAY ZERO

Trusses
Deck

Overhang
Soffits

Flashings

Vapor Barrier
Cladding
Forward Pass

Earliest Times from the Forward Pass Calculation

START
0

0  #1  2
  2
  2

2  #2  9
  7
  7

9  #3  11
  2
  2

#4
9
4

#6
4

#7
6

END
Backwards Pass

Latest Times from the Backward Pass Calculation
Recall: PDM Relationships

- PDM Extends CPM to include
  - Multiple relationships (SS, SF, FF) beyond FS
  - Lags (negative as “leads”)
- Consider relationship XY with lag t between activities A and B
  - X, Y ∈ { S, F }, t ∈ R
  - Interpretation is that event Y of activity B can occur no earlier than t units after evnt X occurs for activity A
  - Think of relationships as linking events
- Special relationships not needed in AOA
  - Can be placed directly between nodes
Notation

- Nodes are no longer simply vertices in graph
  - Arrow on left side of node indicates a start relationship
  - Arrow on right side of node indicates finish relationship
- Non-planar networks may require “jumps”
PDM Activity Relationships

Finish-to-Start **Lead**

Finish-to-Start **Lag**

Start-to-Start **Lag**

Partially Adapted from Kellegeiros, 2003
Finish-to-Finish Lag

Excavate Trench

Lay Pipe

\[ FF = +3 \]

Start-to-Finish Lag

Install Wood Paneling & Base

Install Carpeting

\[ SF = +1 \]

Partially Adapted from Kellegeiros, 2003
PDM Caveats

- Can have different semantics, but same result
- Asymmetries complicate reasoning
- Make sure you understand the meaning of relationships – for the software you use!
- “Lag” and “Lead” lack standard definition
- May have different floats for same activity
  - Start float (LS-ES)
  - Finish float (LF-EF)
  - Arises from successors for these events
PDM Caveats II– Critical Path

- Choices impact critical path!
  - E.g. Finish-to-start vs. Start-to-start
  - Think of critical path as running through events
- Tracing critical path can be difficult
  - Non-critical activity can have critical start/finish
- w/o splitting, can be counter-intuitive (longer duration leads to shorter critical path!)
- Finish-finish constraints with leads can lead to “vanishing” critical path
- How critical path displayed depends on software
“Vanishing Critical Path”
The longer A20 is, the smaller the critical path duration – and quicker can complete!
Equivalent Timing Results

Meaning is different
Critical path may be different
Reasoning about Relationships

- Key Point: PDM relationships often represent relationships between particular parts of an activity. Think about
  - On what portion of an activity the other activity depends
  - On how dependency would change if target activity duration changed
- If unclear, think about unbundling activity
Multiple Relationships

Vs.
Asymmetries

Vs.
Bases for Formal Analysis

A30

A20

Method 1

Method 2

S_B

S_A

A30

A

16MAR04 29MAR04

18

A20

B

15MAR04 26MAR04

10

Legal

Illegal
Distinguishing F-F Interpretations
Non-Binding; A10 Time Unaffected
Distinguishing F-F Interpretations

Binding; A20 Waits for A10
Activity Splitting I: Non-Sequential

- Some algorithms allow division of an activity into two non-sequential pieces
- Advantages: Allows more flexibility in time, resource demands
  - Permits shorter critical paths
  - Eliminates counter-intuitive cases where prefer longer activity
  - Allows predecessor activities connected via SF and SS relationships to begin
  - Allows successor activities connected via SF or FF relationships to begin bulk of work early, and then just wait for event to finish
The longer A20 is, the smaller the critical path duration – and quicker can complete!
Example

- Because of executive offices, can’t *finish* carpeting until wood panelling starts
- Problem: Want carpenters for other work
- Answer: Split wood paneling
  - Do all carpeting except executive offices
  - Allow carpenters to work on executive offices
  - Finish carpeting work for the executive offices
  - Carpenters back to finish job once available
Activity Splitting 2: Pipelining

- Turns monolithic tasks into sub-tasks that operate in parallel
- Typically increases resource demand
- Typically done manually (generally not enough information to permit automation)
- Often represent with S-S constraint

Bubble Patterns when Splitting Tasks
Activity Windows

- Mechanism for imposing time constraints on absolute activity times
- Can impose constraint for any of times
  - ES, EF, LS, LF
  - By set WES=WLS, fix exact timing
- Particularly useful for time-critical milestones
Forward Pass for node k (no splits; no leads)

\[ ES_k = \max_{all\ p} \left\{ \begin{array}{l}
\text{INITIAL TIME} \\
WES_k \\
WEF_k - D_k \\
EF_p + FS_{pk} \\
ES_p + SS_{pk} \\
EF_p + FF_{pk} - D_k \\
ES_p + SF_{pk} - D_k
\end{array} \right. \]

\[ EF_k = ES_k + D_k \]

Key factor: Cannot start until all predecessors ready!
Must take maximum of predecessors’ values
Backward Pass
(node k, no splitting; no leads)

TERMINAL TIME
\[
LF_k = \min_{s} \left\{ WLF_k, WLS_k + D_k, LS_s - FS_{ks}, LF_s - FF_{ks}, LS_s - SS_{ks} + D_k, LF_s - SF_{ks} + D_k \right\}, \quad LS_k = LF_k - D_k
\]

Key factor: Must finish in time for all successors to start in time! Otherwise will delay project completion time => Must take min of successors’ values.
Dealing with Leads

- Dealing with *leads* ("negative lags") requires more general algorithm
  - Two $O(n)$ passes may no longer be sufficient
- Be careful about meaning!
- Basic approach: Convert AON into AOA-like form
  - Start/Finish Nodes explicit for every activity
  - Very helpful for thinking through meaning
- Use Dijkstra’s algorithm to solve $O(V\log V + E)$
Example Translated Diagram
Unified Algorithm

Calculations for the Unified Network Model with Negative Link Durations

**Forward Pass:**

1. **Step 1:**
   - Set $PL(i) = -\infty$ and $TL(i) = -\infty$, where $\infty$ is a number larger than any link duration.
   - Set $TL(PS) = 0$
   - where $PS$ = the project start node
   - $PL(i)$ = the maximum distance from $PS$ to node $i$
   - $TL(i)$ = the maximum distance from $PS$ to node $i$ found at intermediate stages

2. **Step 2:**
   - Select node $i$ for which $TL(i)$ is the maximum among all nodes.
   - Set $PL(i) = TL(i)$ and $TL(i) = -\infty$
   - For each link originating at node $i$,
     - If $PL(j) = -\infty$ and $PL(i) + D(i, j) > TL(j)$, then set $TL(j) = PL(i) + D(i, j)$.
     - If $PL(j) = -\infty$ and $PL(i) + D(i, j) \leq TL(j)$, then do not change the labels on $j$.
     - If $PL(j) > -\infty$ and $PL(i) + D(i, j) > PL(j)$, then set $PL(j) = -\infty$ and $TL(j) = PL(i) + D(i, j)$.
     - If $PL(j) > -\infty$ and $PL(i) + D(i, j) > PL(j)$, then do not change the label on $j$.

3. **Step 3:**
   - Repeat step 2 until $PL(PF) > -\infty$, where $PF$ is the project finish node.

4. **Step 4:**
   - Set the earliest event time for each node, $E(i) = PL(i)$.

**Backward Pass:**

Repeat application of the algorithm with the following changes:

1. Reverse each link direction.
2. Start with the project finish node $PF$ with $TL(PF) = 0$.
3. At the end of step 3, set the latest event time, $L(i) = E(PF) - PL(i)$ for all nodes $i$. 
Motivations for Dealing with Uncertainty

- Schedules exhibit much uncertainty
  - Weather occurrences, design duration, productivity, delivery times, subcontractor quality, regulatory changes, etc...

- Clients, community may want to know milestones, finish date with confidence
  - Tenant move-in dates
  - Traffic planning
  - Event planning

- Reasoning about schedule constraints such as weather, seasonal traffic

- Extensions may be much worse than early completion
Case 1: Logan International Terminal

- Firm date required by
  - Vendors
  - Airlines

- Sought probabilistic scheduling to quantify uncertainty
Case 2: Philadelphia Children’s Hospital

- Described in “Modern Steel Construction” article (posted on STELLAR site)
- Accounting for
  - Emergency helicopter usage
  - Patient area activity
  - Limited time windows for work (2-4am)
  - Contingencies regarding telephone switch relocation
Informal Ways of Handling Uncertainty

- Most common: Ignore!
  - Assume expected duration
  - Hope errors cancel
- Apply contingency factors
- “What if” scenario analysis to examine
  - Optimistic scenario
  - Most likely scenario
  - Pessimistic scenario
Program Evaluation and Review Technique (PERT)

- Developed by US Navy, Booz-Allen Hamilton and Lockheed Corporation
  - Polaris Missile/Submarine (1958)
- Captures probabilistic activity durations
- Allows analytic solution for
  - Schedule duration
  - Schedule variance
PERT Basics

- Beta Distribution for Activity Duration

- Assume normally distributed project duration
  - Project Duration Tends to be Normally Distributed (approx. sum of random variables)
  - Assumes Independent Activity Durations - Not Always Satisfied
Activity Duration Frequency

![Bar chart showing the frequency of activity durations in months. The x-axis represents duration in months (10, 5, 0, 5, 0, 5, 0, 5, 0, 6, 6, 7), and the y-axis represents the number of repetitions (0 to 10). The chart peaks around a duration of 3 months with the highest number of repetitions.](image-url)
Beta Distribution

![Beta Distribution Graph](image_url)
Three Cases of Beta Distribution

- Symmetric
- Skewed to Right
- Skewed to Left
Beta vs. Normal

Can guarantee Beta non-negative
Normal Distribution Assumed for Schedule
Stochastic Approach

- Optimistic
- Most Likely (mode – not mean)
- Pessimistic
- Expected Duration
- Variance
- Standard Deviation

\[ \bar{d} = \frac{1}{3} \left[ 2m + \frac{1}{2} (a + b) \right] = \frac{a + 4m + b}{6} \]

\[ \nu = s^2 \]

\[ s = \frac{b - a}{6} \]
Steps in PERT Analysis

- For each activity $k$
  - Obtain $a_k$, $m_k$ (mode) and $b_k$
  - Compute expected activity duration (mean) $d_k = t_e$
  - Compute activity variance $v_k = s^2$

- Compute expected project duration $D = T_e$ using standard CPM algorithm

- Compute Project Variance $V = S^2$ as sum of critical path activity variance (*this assumes independence!*)
  - In case of multiple critical paths use the one with the largest variance

- Calculate probability of completing the project
  - Assuming project duration normally distributed
<table>
<thead>
<tr>
<th>Activity</th>
<th>Predecessor</th>
<th>a</th>
<th>m</th>
<th>b</th>
<th>d</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2.17</td>
<td>0.25</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>6.00</td>
<td>0.11</td>
</tr>
<tr>
<td>C</td>
<td>-</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>3.83</td>
<td>0.25</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2.83</td>
<td>0.25</td>
</tr>
<tr>
<td>E</td>
<td>C</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>5.17</td>
<td>0.25</td>
</tr>
<tr>
<td>F</td>
<td>A</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4.00</td>
<td>0.11</td>
</tr>
<tr>
<td>G</td>
<td>B,D,E</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2.00</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Activity on Node Example
Forward Pass
Backward Pass
PERT Example-Standard Deviation

\[ T_e = 11 \]


\[ = 0.25 + 0.25 + 0.1111 \]

\[ = 0.6111 \]

\[ S = \sqrt{0.6111} \]

\[ = 0.7817 \]
PERT Analysis - Probability of Ending before 10 (Critical Path Only)

\[ P(T \leq T_d) = P(T \leq 10) \]
\[ = P\left( z \leq \frac{10 - T_e}{S} \right) \]
\[ = P\left( z \leq \frac{10 - 11}{0.7817} \right) \]
\[ = P(z \leq -1.2793) \]
\[ = 1 - P(z \leq 1.2793) \]
\[ = 1 - 0.8997 \]
\[ = 0.1003 \]
\[ = 10\% \]
PERT Analysis - Probability of Ending before 13 (Critical Path Only)

\[
P(T \leq 13) = P\left(z \leq \frac{13 - 11}{0.7817}\right) \\
= P(z \leq 2.5585) \\
= 0.9948
\]
\[ P\left( T_L \leq T \leq T_U \right) = P\left( 9 \leq T \leq 15 \right) \]
\[ = P\left( T \leq 11.5 \right) - P\left( T \leq 9 \right) \]
\[ = P\left( z \leq \frac{11.5 - 11}{0.7817} \right) - P\left( z \leq \frac{9 - 11}{0.7817} \right) \]
\[ = P\left( z \leq 0.6396 \right) - P\left( z \leq -2.5585 \right) \]
\[ = P\left( z \leq 0.6396 \right) - \left[ 1 - P\left( z \leq 2.5585 \right) \right] \]
\[ = 0.7389 - \left[ 1 - 0.9948 \right] \]
\[ = 0.7389 - 0.0052 \]
\[ = 0.7337 \]