Tools for Project Evaluation

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Basic Compounding

- Suppose we invest $x$ in a bank offering interest rate $i$
- If interest is compounded annually, asset will be worth
  - $x(1+i)$ after 1 years
  - $x(1+i)^2$ after 2 years
  - $x(1+i)^3$ after 3 years ….
  - $x(1+i)^n$ after $n$ years
Opportunity Cost &
The Time Value of Money

- If we assume
  - money can always be invested in the bank (or some other reliable source) now to gain a return with interest later
  - That as rational actors, we will never make an investment which we know to offer less money than we could get in the bank

- Then
  - money in the present can be thought as of “equal worth” to a larger amount of money in the future
  - Money in the future can be thought of as having an equal worth to a lesser “present value” of money
Notation

- Cost

- Revenue

- Simple investment

Time
Given a source of reliable investments, we are indifferent between any cash flows with the same present value – they have “equal worth”

This indifferences arises b/c we can convert one to the other with no extra expense
Future to Present Revenue

- Given Future Revenue in year $t$ ...
  
  $r/(1+i)^t$ 
  
- Borrow a smaller amt now from reliable source
  
  $r/(1+i)^t$ 
  
- Transforms future revenue to equivalent present revenue, at no additional cost burden
Future to Present Cost

- Given Future Cost in year $t$...
  \[ \frac{c}{(1+i)^t} \]

- Invest a smaller amt **now** in reliable source
  \[ \frac{c}{(1+i)^t} \]

- Transforms future cost to equivalent present cost, with no additional cost burden
  \[ \frac{c}{(1+i)^t} \]
Present to Future Revenue

- **Present to Future Revenue**
  - Given Present Revenue...
  - Invest all **now** in reliable source; withdraw at t
  - Transforms present revenue to equivalent future revenue, at no additional cost burden
Present to Future Cost

- Given a present cost...
  \[ c \]

- Borrow = amt now from source; pay back at \( t \)
  \[ c \]

- Transforms current cost to equivalent future, with no additional cost burden
  \[ c(1+i)^t \]
Summary

- Given a reliable source offering annual return \( i \)
  we can shift costlessly between cash flow \( v \) at
  time 0 and \( v(1+i)^t \) at time \( t \)

- Because we can flexibly switch from one such
  value to another without cost, we can view these
  values as equivalent

- The present value of a cash flow \( v \) at time \( t \) is
  just \( v/(1+i)^t \)
Notion of Net Present Value

Suppose we had

- A collection (or stream) of costs and revenues in the future
- A certain source of borrowing/saving (at same rate)

The net present value (NPV) is the sum of the present values for all of these costs and revenues

- Treat revenues as positive and costs as negative
Understanding Net Present Value

- NPV (and PV) is relative to a discount rate
  - In our case, this is the rate for the “reliable source”
- NPV specifies the
  - Value of the cash stream beyond what could be gained if the revenues were returns from investing the costs (at the appropriate times) in the “reliable source”
    - The “reliable source” captures the opportunity cost against which gains are measured
- Key point: NPV of “reliable source” is 0
  - PV(revenue from investment) = PV(investment cost)
Example: High-Yield Investment

- Assume reliable source with 10% annual interest
- Invest $100 in high-risk venture at year 0
- Receive $121 back at year 1
- What is the net present value of this investment?
- What is the net future value of this investment?
- What does this mean?

$121

$100
Example: Money in Mattress

- Assume reliable source with 10% annual interest
- Place $100 in mattress at year 0
- Retrieve $100 from mattress at year 1
- What is the net present value of this investment?
- What does this mean?
Discounted Cash Flow

- Computing Present Value (PV) of costs & benefits involves successively discounting members of a cash flow stream
  - This is because the value of borrowing or investment to/from the “reliable source” rises exponentially
- This notion is formalized through
  - Choice of a discount rate \( r \)
    - In the absence of risk or inflation, this is just the interest rate of the “reliable source” (gain through opportunity costs)
    - Applying discount factor \( 1/(1+r)^t \) to values at time \( t \)
Outline

- Session Objective
- Project Financing
  - Public
  - Private
  - Project
  - Contractor
  - Additional issues
- Financial Evaluation
  - Time value of money
  - Present value
  - NPV & Discounted cash flow
    - Simple Examples
    - Formulae
    - IRR
- Missing factors
Develop or not Develop

- Is any individual project worthwhile?

- Given a list of feasible projects, which one is the best?

- How does each project rank compared to the others on the list?

  - “Objective: Strive to secure the highest net dollar return on capital investments which is compatible with the risks incurred”
We can EITHER Use NPV to

- Evaluate a project against some opportunity cost
  - Use this opportunity to set the discount rate $r$

<table>
<thead>
<tr>
<th>NPV</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>Accept the project</td>
</tr>
<tr>
<td>= 0</td>
<td>Indifferent to the project</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>Reject the project</td>
</tr>
</tbody>
</table>

- Use NPV to choose the best among a set of (mutually exclusive) alternative projects
Rates

- Discount Rate:
  - Worth of Money + risk

- Minimum Attractive Rate of Return (MARR)
  - Minimum discount rate accepted by the market corresponding to the risks of a project
Choice of Discount Rate

\[ r = r_f + r_i + r_r \]

Where:

- \( r \) - is the discount rate.
- \( r_f \) - the risk free interest rate. Normally government bond.
- \( r_i \) - Rate of inflation. It is measured by either by consumer price index or GDP deflator.
- \( r_r \) - Risk factor consisting of market risk, industry risk, firm specific risk and project risk.
  - Market Risk
  - Industry Risk
  - Firm Specific Risk
  - Project Risk
Project Evaluation Example

- Warehouse A
  - Construction = 10 months
  - Cost = $100,000/month
  - Sale Value = $1.5M
  - Total Cost?
  - Profit?
  - Better than B?

- Warehouse B
  - Construction = 20 months
  - Cost = $100,000/month
  - Sale Value =$2.8M
  - Total Cost?
  - Profit?
  - Better than A?

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Drawing out the examples

- **Project A**
  - Duration: 10 Months
  - Funding:
    - $100,000
    - $100,000
    - $100,000
    - $100,000
    - ... $100,000
  - Total: $1,500,000

- **Project B**
  - Duration: 20 Months
  - Funding:
    - $100,000
    - $100,000
    - $100,000
    - $100,000
  - Total: $2,800,000
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Interest Formulas

- $i = \text{Effective interest rate per interest period (discount rate of MARR)}$
- $t = \text{Number of compounding periods}$
- $PV = \text{Present Value}$
- $NPV = \text{Net Present Value}$
- $FV = \text{Future Value}$
- $A = \text{Annuity}$
Interest Formulas: Payments

- **Single Payment Compound Amount Factor**
  
  \((F/P, i\%, n) = (1 + i)^n\)

- **Single Payment Present Worth Factor**
  
  \((P/F, i\%, n) = 1 / (1 + i)^n = 1 / (F/P, i\%, n)\)

- **Uniform Series Compound amount Factor**
  
  \((F/A, i\%, n) = (1 + i)^n - 1 / i\)

- **Uniform Series Sinking Fund Factor**
  
  \((A/F, i\%, n) = i / (1 + i)^n - 1 = 1 / (F/A, i\%, n)\)

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Interest Formulas: Series

- **Uniform Series Present Worth Factor**

  \[(P/A, \ i\%/\, n) = \frac{1}{i} - \frac{1}{i (1 + i)^n}\]

- **Uniform Series Capital Recovery Factor**

  \[(A/P, \ i\%/\, n) = \frac{[i (1 + i)^n]}{[(1 + i)^n - 1]} = \frac{1}{(P/A, \ i\%/\, n)}\]

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Note on Continuous Compounding

- To this point, we have considered annual compounding of interest.

- Consider more frequent compounding:
  - Interest is in %/year.
  - Fraction of interest gained over time $\Delta t$ (measured in years) $= i \Delta t$.
  - For $n$ compounding periods/year, effective rate for entire year is
    $$\left(1 + \frac{i}{n}\right)^n$$
  - As $n \to \infty$ we approach continuous compounding and quantity approaches $e^i$.
  - Over $t$ years, we have $e^{it}$. 

Equipment Example

- $20,000 equipment expected to last 5 years
- $4,000 salvage value
- Minimum attractive rate of return 15%
- What are the?
  - A - Annual Equivalent
  - B - Present Equivalent

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Equipment Example

(a) A = ?
(b) P = ?

P

$20,000

A

1 2 3 4 5

i = 15%

$4,000
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Internal Rate of Return (IRR)

- Identifies the rate of return on an investment
  - Example: Geometrically rising series of values
- Typical means of computing: Identify the discount rate that sets NPV to 0
IRR Investment Rule

\[
\begin{array}{c|c|c}
> & r^* & \text{Accept} \\
= & r^* & \text{Indifferent} \\
< & \text{Reject} \\
\end{array}
\]

“Accept a project with IRR larger than the discount rate.”

Alternatively,

“Maximize IRR across mutually exclusive projects.”

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**Internal Rate-of-Return Method (IRR) Example**

<table>
<thead>
<tr>
<th>MACHINE A</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$20,000</td>
</tr>
<tr>
<td>Life</td>
<td>5 years</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$4,000</td>
</tr>
<tr>
<td>Annual receipts</td>
<td>$10,000</td>
</tr>
<tr>
<td>Annual disbursements</td>
<td>$4,400</td>
</tr>
</tbody>
</table>

- \[ 0(r\%) = -20,000 + 5,600 \left( \frac{P}{A, \ r\%, \ 5} \right) + 4,000 \left( \frac{P}{F, \ r\%, \ 5} \right) \]

- \[ i = +/- 16.9\% > 15\% \text{ then the project is justified} \]
Internal Rate-of-Return Method (IRR) Graph

- Net Present Value ($)
- Interest Rate, $i$ (%)
IRR vs. NPV

- Most times, IRR and NPV give the same decision / ranking among projects.
- IRR does not require to assume (or compute) discount rate.
- IRR only looks at rate of gain – not size of gain.
- IRR ignores capacity to reinvest.
- IRR may not be unique (payments in lifecycle):

Trust NPV: It is the only criterion that ensures wealth maximization

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Other Methods I

- **Benefit-Cost ratio (benefits/costs) or reciprocal**
  - Discounting still generally applied
  - Accept if <1
  - Common for public projects
  - Does not consider the absolute size of the benefits
  - Can be difficult to determine whether something counts as a “benefit” or a “negative cost”

- **Cost-effectiveness**
  - Looking at non-economic factors
  - Discounting still often applied for non-economic
    - $/Life saved
    - $/QALY
Other Methods II

- **Payback period ("Time to return")**
  - Minimal length of time over which benefits repay costs
  - Typically only used as secondary assessment

- **Drawbacks**
  - Ignores what happens after payback period
  - Does not take discounting into account

- **Discounted version called “capital recovery period”**

- **Adjusted internal rate of return (AIRR)**
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What are we Assuming Here?

- That only quantifiable monetary benefits matter
- Certain knowledge of future cash flows
- Present value (discounting) using equal rates of borrowing/lending
Money is not Everything

- Social Benefits
  - Hospital
  - School
  - Employment opportunities
- Intangible Benefits
  - New cafeteria
- Strategic benefits
  - Partnering with firm for long-term relationship
We are missing critical uncertainties

- Revenue
  - Level of occupancy
  - Elasticity and Level of cost
  - Duration of project
  - Post-construction revenue
    - Sale of building

- Costs
  - Construction costs
    - Environmental conditions
    - Labor costs
    - Size of lowest bid
    - Variable interest rates
  - Maintenance costs
    - Energy costs
    - How quickly items wear out
    - Labor costs