Lecture 27

Introduction: Energy bounds in linear elasticity
I. Dimensional analysis
   1. On monsters, mice and mushrooms
   2. Similarity relations: Important engineering tools

II. Stresses and strength
   3. Stresses and equilibrium
   4. Strength models (how to design structures, foundations.. against mechanical failure)

III. Deformation and strain
   5. How strain gages work?
   6. How to measure deformation in a 3D structure/material?

IV. Elasticity
   7. Elasticity model – link stresses and deformation
   8. Variational methods in elasticity

V. How things fail – and how to avoid it
   9. Elastic instabilities
   10. Plasticity (permanent deformation)
   11. Fracture mechanics

Lectures 1-3
Sept.

Lectures 4-15
Sept./Oct.

Lectures 16-19
Oct.

Lectures 20-31
Oct./Nov.

Lectures 32-37
Dec.
I. Dimensional analysis

II. Stresses and strength

III. Deformation and strain

IV. Elasticity
- Lecture 20: Introduction to elasticity (thermodynamics)
- Lecture 21: Generalization to 3D continuum elasticity
- Lecture 22: Special case: isotropic elasticity
- Lecture 23: Applications and examples
- Lecture 24: Beam elasticity
- Lecture 25: Applications and examples (beam elasticity)
- Lecture 26: ... cont’d and closure
- Lecture 27: Introduction: Energy bounds in linear elasticity (1D system)
- Lecture 28: Introduction: Energy bounds in linear elasticity (1D system), cont’d
- ...

V. How things fail – and how to avoid it
- Lectures 32..37
Convexity of a function

\[
\frac{\partial f}{\partial x} \bigg|_{x=a} (b - a) \leq f(b) - f(a)
\]
Example system: 1D truss structure

We will use this example to illustrate all key concepts
Total external work

\[ W^d = \vec{\xi} \cdot \vec{F}^d + \vec{\xi}^d \cdot \vec{R} \]

- Work done by prescribed forces
- Displacements unknown
- Work done by prescribed displacements, force unknown
Total internal work

State equations

\[
N_i = \frac{\partial \psi_i}{\partial \delta_i}
\]

\[
\delta_i = \frac{\partial \psi^*_i}{\partial N_i}
\]

Complementary free energy

\[
\psi_i^* = \psi_i(N_i)
\]

Free energy

\[
\sum_i \delta_i N_i = \psi_i^*(N_i) + \psi_i(\delta_i)
\]
Combining it…

\[ W^d = \tilde{\xi} \cdot \tilde{F}^d + \tilde{\xi}^d \cdot \tilde{R} = \psi + \psi^* \]

\[-(\psi^* - \tilde{\xi}^d \cdot \tilde{R})^! = \psi - \tilde{\xi} \cdot \tilde{F}^d \]

- Complementary energy
  \[ \Rightarrow \mathcal{E}_{\text{com}} \]
- Complementary energy
  \[ \Rightarrow \mathcal{E}_{\text{pot}} \]

Solution to elasticity problem

\[- \mathcal{E}_{\text{com}} = \mathcal{E}_{\text{pot}} \]
Quiz II – Monday Nov. 19

- Focus on material presented in lectures 16-26

**Preparation:** Problem sets, old quizzes, lecture material

- Deformation and strain, isotropic elasticity, beam deformation (beam bending and beam stretching), forensic beam elasticity, sketch solution of beam problems, concept of superposition (frame structures)