Example 1 (Flexural Strength of a Given Member)

\[ b = 12'' \]
\[ d = 17.5'' \]
\[ A_s = 4.00 \text{ in}^2 \]
\[ f_y = 60,000 \text{ psi} \]
\[ f'_c = 4000 \text{ psi} \]

Find \( M_n, M_u \)

\[ M_u \leq \phi M_n; \quad \phi = 0.9 \text{ for flexure} \]

Therefore, with \( M_n, M_u \) can be calculated

\[ M_n = \rho f_y bd^2 \left( 1 - \frac{\rho f_y}{1.7 f'_c} \right) \quad \rightarrow \quad \text{Equation (1)} \]

\[ \rho = \frac{A_s}{bd} = \frac{4.00}{12 \times 17.5} = 0.019 \quad \rightarrow \quad \text{Equation (2)} \]

Check \( \rho_{\text{min}} = \frac{200}{f_y} = 0.0033 \quad \rightarrow \quad \text{Equation (3)} \]

\[ \frac{3}{4} \rho_b = \frac{3}{4} \cdot \frac{0.85 \beta_f f'_c}{f_y} \cdot \frac{87000}{87000 + f_y} = 0.0214 \quad \rightarrow \quad \text{Equation (4)} \]

\[ \therefore \rho_{\text{min}} \leq \rho \leq \frac{3}{4} (\rho_b) \quad \rightarrow \quad \text{Equation (5)} \]

\[ \therefore M_n = 3487 \text{ kips.in} \text{ and } M_u = 0.9 \times M_n = 3138 \text{ kips.in} \]
Example 2 (Section Design with a Given Moment)

Unknowns: \( b, d, h, A_s \)

Given:
- \( l = 15 \text{ feet} \)
- \( DL = 1.27 \text{ kips/ft} \)
- \( LL = 2.44 \text{ kips/ft} \)
- \( f'_c = 4000 \text{ psi} \)
- \( f_y = 60,000 \text{ psi} \)
- \( \gamma_c = 150 \text{ psf} \)

1. Assume \( b \) and \( h \) for self-eighth determination:

   Let \( b = 10 \text{ in} \) and \( h = 18 \text{ in} \)
   
   \[ d = 18 - 2.5 = 15.5 \]
   
   Minimum depth for simply supported beam = \( l/16 = 15/16 \cdot 12 = 11.25 \); OKAY!

2. Find the applied moment to be resisted

   \[ W = 150 \cdot (10/12) \cdot (18/12) \cdot (1/1000) = 0.1875 \text{ kips/ft} \] (this is to be revised)

   Therefore, \( W_u = 1.4(1.27 + 0.1875) + 1.7(2.44) = 6.19 \text{ kips/ft} \)

   \[ M_u = w_u l^2/8 = 6.19 (15)^2/8 \cdot 12 (\text{ft/in}) = 2089 \text{ kips.in} \]

3. Compute \( \rho_{\text{min}} \) & \( \rho_b \); and choose \( \rho \)

   \[ \rho_{\text{min}} = 200/f_y = 0.0033 \]
   
   \( \frac{1}{4} \rho_b = 0.0214; \) use \( \rho = 0.0214 \) (Not economical, but adequate for demonstration purpose)

   Find the required \( bd^2 \)

   \[ M_u = \phi f_y (bd^2) \left( 1 - \frac{\rho f_y}{1.7 f'_c} \right) \]

   \[ bd^2 = 2229 \text{ in}^2 \]

   Actual \( bd^2 = 10 \cdot (15.5)^2 = 2403 \text{ in}^2 > \) Required \( bd^2 = 2229 \text{ in}^2 \); OKAY!

4. Assign rebar arrangement

   \[ \rho = A_s/bd = 0.0214 \]

   \[ A_s = 0.0214 \cdot b \cdot d = 0.0214 \cdot 10 \cdot (15.5) = 3.32 \text{ in}^2 \] (required)

   Provide 2#10 + 1#8

   \( \therefore A_s, \) provide = 3.32 \text{ in}^2

Note: \( \rho \) can be smaller and a larger section may be needed to improve cost and deflection performance. However, if there is architectural restrictions on sizes, a \( \rho \) with a value closer to the upper bound is normally used (to reduce section size as much as possible)
Example 3 (Crack Width Determination)

Given:
- \(b = 12''\)
- \(h = 20''\)
- \(A_s = 4.00 \text{ in}^2 (4 \#9)\)
- \(f_y = 60,000 \text{ psi}\)
- Exposure = external

\[
w = 0.000091 f_y \sqrt{d_c A} \quad \rightarrow \quad \text{Equation 1}
\]

\[f_s = 0.6 f_y \text{ in kips} = 0.6 \times 60 = 36 \text{ kips/in}\]

\[d_c = 2.5 \text{ in}\]

\[A = \frac{A_{\text{eff}}}{N} \quad \rightarrow \quad \text{Equation 2}\]

\[A_{\text{eff}} = \text{web width} \times 2 \times d_c = 12 \times 2 \times 2.5 = 60 \text{ in}^2\]

\[N = \text{Total } A_s / \text{area of largest bar} = \frac{4.00}{1.00} = 4\]

Therefore, \(A = 60 / 4 = 15 \text{ in}^2\)

**W = 0.011 in**

ACI Stipulation
- External exposure: \(W_{\text{max}} = 0.013 \text{ in}\)

Since, \(W < W_{\text{max}}, \text{ OKAY!}\)
Example 4 (Neutral Axis Location of Cracked Section)

Given:
\[ E_c = 3,625,000 \text{ psi} \]
\[ E_s = 29,000,000 \text{ psi} \]

\[ A_s' = 1.2 \]
\[ A_s = 3 \]

\[ n = \frac{E_s}{E_c} = 8 \]

Locate the neutral axis by using \( C = T \)

**Transformed Section**

\[ Y (10) Y/2 + (n-1) A_s' (Y-3) = nA_s (17-5) \]
\[ 5Y^2 + 7(1.2)(Y-3) = 8(3)(17-Y) \]
\[ Y = 6.62 \text{ in} \]
Example 5 (Moment of Cracked Section)

\[ I_{cr} = \frac{bY^3}{12} + (n-1)A_s(Y-3)^2 + nA_s(17-Y)^2 = 10(6.62)^3/3 + 7(1.2)(6.62 - 3)^2 + 8(3)(17-6.62)^2 = 3663 \text{ in}^4 \]

Example 6 (Evaluate Stirrup Spacing for Different Shear Loads)

Given

- \( f'_c = 3000 \text{ psi} \)
- \( f_y = 60,000 \text{ psi} \)
- \( A_v = 0.22 \text{ in}^2 \) (#3 stirrup) \( \rightarrow \Pi(3/8)^2/4 \times 2 \) legs
- \( b = 30'' \)
- \( d = 16.5'' \)
- \( b_w = 10'' \)
- \( \phi = 0.85 \) for shear design

Case 1: \( V_u = 12 \text{ kips} \)
Case 2: \( V_u = 36 \text{ kips} \)
Case 3: \( V_u = 42 \text{ kips} \)

Equations to be used:

- \( V_c = 2\sqrt{f'_c b_w d} \)
- \( s = \frac{d}{2}; \quad s = \frac{A_v f_y}{50 b_w} \)
- \( V_s = \frac{V_u}{\phi} - V_c \)
- \( \phi V_c/2 = 0.85 \times 18.1 / 2 = 7.7 \text{ kips} \)

\[ V_c = 2 (3000)^{1/2} \cdot 10 \cdot (16.5) = 18.1 \text{ kips} \]
Case 1

\[ V_u > \phi V_c / 2 \; \text{since} \; 12 > 7.7 \]

But

\[ V_u < \phi V_c \; \text{since} \; 12 < 15.4 \]

Therefore, use minimum reinforcement, spacing is the smallest of

\[ s = d/2 = 16.5/2 = 8.25" \]

Choose \( s = 8.25" \) \( \rightarrow \) theoretical

Provide \( s = 8" \) \( \rightarrow \) practical

Case 2

\[ V_u > \phi V_c \; \text{since} \; 36 > 15.4 \]

\[ V_s = V_u / \phi - V_c = 36/0.85 - 18.1 = 24.3 \text{ kips} \]

and

\[ s = A_v f_y d / V_s = 0.22 x 60 x 16.5 / 24.3 = 8.96" \]

But need to check if \( V_s \leq 4 \sqrt{f_c'} b_w d = 4 (3000)^{1/2} \times 10 \times (16.5) = 36.1 \text{ kips} > 24.3 \)

Therefore, \( s_{\text{max}} = A_v f_y / 50b_w = 26.4" \) or

\[ s_{\text{max}} = d/2 = 8.25" < 24" \]

\( \rightarrow \) \( s = 8.25" \)

Therefore, provide \( s = 8" \) as before

For \( V_u = 12 \) or 36 kips

Provide the same stirrup arrangement!!!

Case 1: For safety reason

Case 2: For need-based reason

Case 3

\[ V_u = 42 \text{ kips} \]

\[ V_s = V_u / \phi - V_c = 42/0.85 - 18.1 = 31.3 \text{ kips} < 4 \sqrt{f_c'} b_w d = 36.1 \text{ kips} \]

Therefore, provide

\[ s = \frac{A_v f_y d}{V_s} = \frac{0.22 \times 60 \times 16.5}{31.3} = 6.96" \]

Need to check

\[ s_{\text{max}} = \frac{A_v f_y}{50b_w} \text{ or } \frac{d}{2} \text{ or } 24" \]

Therefore, \( s = 6.96" \) controls

Provide \( s = 6.5" \)
Example 7 (Determine Maximum Load Based on Shear Design)

Given:

\[ b = 16" \]
\[ d = 18" \]
\[ L = 20' \]
\[ f_y = 60,000 \text{ psi} \]
\[ f'_c = 3000 \text{ psi} \]
\[ \phi = 0.85 \]

Region 1

Check \( V_c \)

\[ V_c = 2\sqrt{f'_c} b d = 2 \times (3000)^{1/2} \times 16 \times 18 = 31.55 \text{ kips} \]

Find \( V_s \)

\[ V_s = A_v f_y d / s = 0.22 (60) (18) / 4 = 59.4 \text{ kips} \]

Find \( V_u \)

\[ V_u = \phi (V_c + V_s) = 0.85 (31.55 + 59.4) = 77.3 \text{ kips} \]

Find \( W_u \) allowed

At the critical section \( V_u = 8.5 W_u \)
Therefore, \( W_u = 77.3 = 9.1 \text{ kips/ft} \)
Region 2

\( V_c = 31.55 \text{ kips (same)} \)

\( V_s = A_v f_y d / s = 59.4 \times (4/9) = 26.4 \text{ kips} \)

\( V_u = \phi (V_c + V_s) = 0.85 (31.55 + 26.4) = 49.3 \text{ kips} \)

Find \( W_u \) allowed

At the transition, the interface will be taken care of by the last stirrup in Region (1). Therefore, consider \( d \) from the interface.

\( V_u = 3.5 W_u = 49.3 \)

\( W_u = 14.1 \text{ kips/ft} \)

Obviously Region (1) controls

\( W_u (\text{max}) = 9.1 \text{ kips/ft} \)
Example 8 (Design of Stirrups with Moment-Shear Coupling Consideration)

Given:

\[
\begin{align*}
\text{Load} & = 8 \times 20 / 2 = 80 \text{ kips} \\
\text{DL} & = 1.45 \text{ kips/ft (Include Self-Weight)} \\
\text{LL} & = 3.5 \text{ kips/ft} \\
A_s & = 6.06 \text{ in}^2 \\
f_c' & = 2500 \text{ psi} \\
f_y & = 50,000 \text{ psi} \\
b & = 16'' \\
d & = 22''
\end{align*}
\]

1. Compute factored load

\[
W_u = 1.4 \times \text{DL} + 1.7 \times \text{LL} = 1.4 \times 1.45 + 1.7 \times 3.5 = 7.98
\]

Use \(W_u = 8.0\) kips/ft

2. Compute \(M_u, V_u\) at critical section, \(d\) from the support

\[
\begin{align*}
d & = 22' = 22/12 \\
V_u & = 80 - 8(22/12) = 65.3 \text{ kips} \\
M_u & = (80 + 65.3) / 2 \times (22/12) = 133.19 \text{ kips.ft}
\end{align*}
\]

3. Compute nominal shear strength
\[ V_c = \left( 1.9 \sqrt{f'_c} + 2500 \rho_w \frac{V_u d}{M_u} \right) b_u d \leq 3.5 \sqrt{f'_c} b_u d \]

Therefore, \( V_u d/M_u = 65.3 \times 22 / (133.19 \times 12) = 0.9 < 1.0; \) \text{ OKAY!} \\
\rho_w = \frac{A_s}{b_u d} = 6.06 / (16 \times 22) = 0.0172 \\
V_c = 47.06 \text{ kips} \\
Check \( 3.5 \sqrt{f'_c} b_u d = 61.6 \text{ kips} > V_c; \text{ OKAY!} \)

4. Stirrup provision
\[ \phi V_c / 2 = 0.85 (47.06) / 2 = 20 \text{ kips} < V_u \]
Thus, need stirrups
\[ V_s = V_u / \phi - V_c = 65.3/0.85 - 47.06 = 29.76 \text{ kips} \]
\[ s = A_s f_y d / V_s = 0.22 \times 50 \times 22 / 29.76 = 8.13" \]
Check \( s_{\text{max}} \)
\[ \text{i.} \quad d/2 = 11 \text{ in.} \]
\[ \text{ii.} \quad 4 \sqrt{f'_c} b_u d = 70.4 \text{ kips} > V_s \]
\[ s_{\text{max}} = A_s f_y / 50 b_w \text{ or } d/2 \text{ or } 24" \]
\[ = 0.22 \times 50,000 / (50 \times 16) = 13.75 \]
Therefore, \( s = 8.13" \) controls
**Use \( s = 8" \)**

5. Determine where to terminate by computing \( V_c \) and check against \( \phi V_c / 2 \)
Example 9

Determine the $M_n$ for the given section
(a) $b_E = L/4 = 26/4 \times 12 = 78''$
(b) $b_E = b_w + 16t = 13 + 16 \times 4.5 = 85''$
(c) $b_E = 12 \times 13 = 156''$

if $a = 4.5'' = t$

$c = 0.85 \times f_c' \times b_E \times a = 0.85 \times 3000 \times 78 \times 4.5 = 895$ kips

For equilibrium

$C = T = A_s f_y$

$A_s = 895,000/50,000 = 17.9$ in$^2$ (required)

Steel reinforcement provided = 6.28 in$^2 < 17.9$ in$^2$; Therefore, $a < t$

This means that we should design according to a rectangular beam (simply reinforced)

$M_n = T \times (d - a/2)$

$T = A_s f_y = 6.28 \times 50 = 314$ kips

$A = T / (0.85 \times f_c' \times b_E) = 314 / (0.85 \times 3 \times 78) = 1.58''$
Example 10 (Determine $M_n$ with the given section (isolated))

\[ A_s = 12.48 \text{ in}^2 \]
\[ b_E = 30'' \]
\[ b_w = 14'' \]
\[ d = 36'' \]
\[ t = 7'' \]
\[ f'_c = 3000 \text{ psi} \]
\[ f_y = 50,000 \text{ psi} \]

Check:
\[ 4b_w = 56'' > b_E; \quad \text{OKAY!} \]
\[ \frac{1}{2} b_w = 7'' > t; \quad \text{OKAY!} \]

If $a = t = 7''$
\[ C = T \rightarrow 0.85 f'_c b_w a = 0.85 f_c x 30 x 7 = 535.5 = A_s f_y \]
\[ A_s = 10.71 < 12.48 \text{ (provided)}; \]
Therefore, $a > t$ (i.e. Neutral axis is below the flange)

\[ C_1 = 0.85 f'_c b_w a = 0.85 x 3 x 14 x a = 35.7a \]
\[ C_2 = 0.85 f'_c (b_E - b_w) t = 0.85 x 3 x 930-14) x 7 = 285.6 \]

\[ T = A_s f_y = 12.48 x 50 = 624 \]
Therefore,
\[ 624 = 35.7a + 285.6 \\
\frac{a}{a} = 9.48'' \]

\[ M_n = C_1(d-a/2) + C_2(d-t/2) = 35.7 x 9.48 x (36 - 9.48/2) + 285.6 x (36 - 7/2) = 1155 \text{ kips.ft} \]
Example 11 (Design t-Beam with Given Load)

\[ \text{DL} = 370 \text{ kips.ft} \quad b_e = 30'' \]
\[ \text{LL} = 520 \text{ kips.ft} \quad b_w = 14'' \]
\[ f'_c = 3000 \text{ psi} \quad d = 36'' \]
\[ f_y = 50,000 \text{ psi} \quad t = 7'' \]

\[ M_u = 1.4 \text{DL} + 1.7 \text{LL} = 1.4 \times 370 + 1.7 \times 520 = 1402 \text{ kips.ft} \]
\[ M_n = M_u/\phi = 1402/0.9 = 1557.8 \approx 1560 \text{ kips.ft} \]

Find position of neutral axis (NA)

If \( a = t \)
\[ T = C = 0.85 \times 3 \times 30 \times 7 = 535 \text{ kips} \]
\[ M_n = C(d - a/2) = 535(36 - 7/2) = 1450 \text{ kips.ft} < 1560 \text{ (required)} , \]

Therefore, \( a > t \)

\[ C_1 = 0.85 \times 3 \times 14 \times a = 35.7a \]
\[ C_2 = 0.85 \times 3 \times (30-14) \times 7 = 285.6 \]

\[ T = 0.85 f'_c b_w a + C_2 = 0.85 \times 3 \times 14 \times 8.3 + 285.6 = 582 \text{ kips} \]
\[ A_s = 582 / f_y = 11.64 \text{ in}^2 \]

\[ a_b = 0.85 \left( \frac{0.003}{0.003 + \frac{50}{29000}} \right) \times 36 = 19.4 \text{in}^2 \]

\[ A_{s1b} = (0.85 f'_c b_w a_b) / f_y = 13.85 \]
\[ A_{s2b} = (0.85 f'_c (b_e - b_w) t) / f_y = 5.71 \]

Therefore, \( A_{s,max} = 0.75 (A_{s1b} + A_{s2b}) = 14.7 \text{ in}^2 > 11.64 \text{ in}^2; \quad \text{OKAY}!! \)