Recitation 5 – Part I
Design Procedures of Short Columns
In Accordance to the ACI-318

Reference: Chapter 10, Building Code Requirements for Reinforced Concrete ACI-318 and Commentary (ACI-318R), American Concrete Institute, Box 19150, Redford Station, Detroit, Michigan 48219

The ACI Procedure for Classifying Beam-Columns
A column can be classified as short (stocky) when its effective slenderness ratio satisfies the following criteria:

\[
\frac{k \cdot l_u}{r} \leq 22 \quad \text{for unbraced members}
\]

\[
\frac{k \cdot l_u}{r} \leq 34 - 12 \frac{M_1}{M_2} \quad \text{for braced members}
\]

where

- \( k \) = effective-length factor (see Appendix)
- \( l_u \) = unsupported length of member; clear distance between floor slabs, beams, or other members capable of providing lateral support
- \( r \) = radius of gyration of cross section of column associated with axis about which bending is occurring
  - \( = 0.3 \) (depth of section in direction of bending) for rectangular section
  - \( = 0.25 \) (section diameter) for circular section
- \( M_1 \) = smaller end moment on the column
- \( M_2 \) = larger end moment on the column

A study found that ~90% of the columns in braced frames and >40% of the columns in unbraced frames were short (stocky). Since most structural frames are braced, secondary moments will usually be small and can be neglected in the design of most columns. That’s also the reason why short column design is emphasized in this course.

A Note on Short Column Design
For columns that are components of rigid-jointed structures, the analysis and design are inter-related. Since the distribution of forces in an indeterminate structure depends on the relative stiffness of the members, the dimensions of members must be estimated before the structure can be analyzed. After both the axial load and the maximum moment in the column due to factored loads are determined, the reinforcement is selected. If the area of
steel falls within the 1 – 8 percent limits established by the ACI-318, the design is complete, otherwise, the dimensions must be readjusted and the analysis and design repeated.

ACI Clause 10.10.1 indicates an analysis is acceptable if the difference between the assumed dimensions and the final dimensions does not exceed 10 percent. For instance, if a designer initially assumes a column with 10" square section in a particular location in a building frame will be adequate to carry floor loads, but the calculated loads from the analysis indicate a 12" section is required, the designer must repeat the analysis using the properties of the larger section.

For most building frames, small changes in column dimensions do not produce significant changes in the column forces; there, an acceptable column section can usually be established in one or two trials. To complete the design, the ties or spirals enclosing the steel are sized and the shear strength of the cross section checked. Shear forces are typically small in interior columns, but may be substantial in exterior columns bent into double curvature.

Design Methods

1. Interaction Diagrams (Square or circular sections)
2. Trial Method (All sections)

1. Interaction Diagrams

The dimensions of a column cross section and the required area of reinforcement required to support a specific combination of axial load and moment due to factored loads can be quickly established by using interaction curve handbooks. Some examples are given in the Appendix. Both tied and spiral columns with common patterns of reinforcements are usually contained. To permit these interaction curves to be used for cross sections of any dimension, the ordinates are plotted in terms of \( \frac{\Phi P_n}{A_g} \) and the abscissas in terms of \( \frac{\Phi M_n}{A_g} \).

Step 1: Select the properties of the materials \( f'_c \) and \( f_y \) and the parameter \( \gamma \), a function of the distance between rows of reinforcement. Select a trial area \( A_g \).

Step 2: Determine the appropriate design aid (interaction curve).

Step 3: Compute the coordinates required to enter the design aid.

The section axial strength \( \Phi P_n \) is set equal to \( P_u \); \( e \) is computed by \( M_u/P_u \).
Step 4: Read the required value of the reinforcement ratio $\rho_g$ from the curve that lies at the intersect of the coordinates.

Step 5: Compute the total area of steel, $A_{st}$, from $A_{st} = \rho_g A_g$.

Step 6: Select bars, check clearances, and size ties or spirals to complete design.

2. Trial Method

When an interaction curve is not available because a cross section has an unusual shape or a nonstandard pattern of reinforcement, cross sections can be designed by a method of trials. The procedure is set forth below:

Step 1: Select a trial section and analyze the structure to establish the design forces $P_u$ and $M_u$ produced by factored service loads. An approximate trial area can be established by dividing $P_u$ by 0.5 $f'_c$ when the eccentricity does not exceed half the depth of the section. For larger eccentricities use 0.3 to 0.4 $f'_c$.

Step 2: Establish a reinforcement pattern. The area of steel $A_{st}$ might be taken initially as 3 or 4 percent of the gross area of the cross section.

Step 3: Compute the eccentricity of the axial force $e = \frac{M_u}{P_u}$.

Step 4: Guess a strain distribution associated with failure, compute the internal forces, and solve for $P_n$ and $M_n$ using vertical and moment equilibriums.

Step 5: Compute $e = \frac{M_u}{P_u}$ and $\Phi P_n$. If $P_u < \Phi P_n$ and the eccentricity of $P_n$ is equal to or slightly greater than the eccentricity of $P_u$, the cross section is adequate. If a larger difference in eccentricity exists between $P_u$ and $P_n$, modify the strain distribution and repeat the analysis. If the axial strength of the section $\Phi P_n$ is too large or too small, modify the cross section by changing the area of steel or the dimensions, and repeat the analysis and design.
Design of Spirals and Ties

A. Spiral
- Bar size $\geq $ #3
- Clear spacing $3'' \geq S \geq 1''$

B. Ties
- Bar size $\geq $ #3 if longitudinal bars $\leq $ #10
- Bar size $\geq $ #4 if the otherwise including bundled bars
- Spacing $\leq$ the smallest of the following:
  a) $\frac{1}{16}$ (longitudinal bar diameter)
  b) 48 (tie diameter)
  c) Least dimension of the section, b
- Every corner and alternate longitudinal bar has lateral support provided by the corner of a tie having an included angle of not more than 135 degree
- No bar shall be farther than 6” clear on either side from such a laterally supported bar

Rules of Thumb
- $A_{st} \geq 1\% \cdot A_g$ as always, to provide ductility, reduce creep and shrinkage, ensure some bending strength due to accidental eccentricity
- $A_{st} \leq 5\% \cdot A_g$ for sufficient space for concrete flow between bars during casting
- Use bundled bars if $5\% \cdot A_g \leq A_{st} \leq 8\% \cdot A_g$ to prevent honeycomb and voids that would reduce column strength significantly
- Bar spacing need to be greater than $1.5\*(\text{bar diameter})$ or $1.5''$ whichever is greater
- Cover need to be greater than $1.5''$ for interior exposure; $2.0''$ for exterior exposure
- Use minimum section dimension $\geq 10''$ for construction clearance