1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 10

Torsion, Shear, and Flexure

- Torsion
  - Stress distribution on a cross section subject to torsion

\[
\tau_{\text{max}} = \eta \frac{T}{x^2y}
\]

where \( \eta \) = shape factor, \( T \) = torque, \( x, y \) = dimensions of the cross section. The shape factor is different for linear and nonlinear cases.
Failure mode

- Torsion failure of plain concrete occurs suddenly with an inclined tension crack in one of the wider faces, then extending into the narrow faces. Concrete crushing occurs in the opposite wider face.

Torsional strength, $T_{up}$, of plain concrete

- Several theories have been presented for computing torsional strength of plain concrete including elastic, plastic, and skew bending theories.
- Skew bending:

\[ T = \frac{T}{1} \text{, } b = \sqrt{2} \cdot y, \]

\[ S = \frac{\sqrt{2}y}{6} = \frac{x^2y}{3\sqrt{2}} \]

\[ \sigma_t = \frac{M}{S} = \frac{T_{up}}{\sqrt{2}x^2y} = \frac{3T_{up}}{x^2y} \]

where $T_{up}$ = ultimate torsion for plain concrete when $\sigma$ reaches $\sigma_t$.

\[ T_{up} = \frac{x^2y}{3} \sigma_t \]

\[ \sigma_t = 0.85f_r \approx 0.85 \left( 7.5\sqrt{f_c} \right) = 6\sqrt{f_c} \]
\[ T_{cr} = \frac{x^2 y}{3} \cdot 6 \sqrt{f_c} = 2x^2 y \sqrt{f_c} = T_{up} \]

- Torsional strength contributed by steel
  - Consider the system consisting of longitudinal and transverse (stirrups) steel:
    \((x_i, y_i)\) are the dimensions of steel frame as shown.)

- Torsional moment with respect to axis of the vertical stirrups
  \[ T_{s1} = \left( A_s f_s \right) \frac{y_i}{s} x_i \]
  where \( A_s \) = area of one stirrup leg,
  \( f_s \) = stirrup stress, and
  \( s \) = stirrup spacing.

- Torsional moment with respect to axis of the horizontal stirrups
  \[ T_{s2} = \left( A_s f_s \right) \frac{x_i}{s} y_i \]

- Total torsional moment
  \[ T_s = \frac{A_s f_c}{s} x_i y_i \left( \alpha_1 + \alpha_2 \right) \]
  \[ T_s = \frac{A_s f_c}{s} x_i y_i \alpha_i \] (\( \alpha_i \) is determined from experiment.)
Design concept

- Total ultimate torsion capacity, $T_u$
  
  \[ T_u = T_c + T_s \]

  where $T_c = \text{torsional capacity contributed by concrete}$, and $T_s = \text{torsional capacity contributed by reinforcement}$.

  \[ T_c = \beta T_{up} \quad (\beta \approx 0.4) \]

  Thus,

  \[ T_c = 0.8 \sqrt{f_c' x^2 y} \]

  The coefficient $\beta$ represents reduction in torsional strength provided by concrete after cracking. Upon cracking of concrete stress and strain are partially transferred to steel. Stiffness and strength of the system will depend on the amount of transverse and longitudinal reinforcements.

- The final failure may be in one of the following ways:
  1. Under reinforced $\rightarrow$ Both transverse and longitudinal steel yield before failure.
  2. Over reinforced $\rightarrow$ Concrete crushes before yielding of steel.
  3. Partially over (under) reinforced

- For under reinforced elements, $\alpha_t$ is independent of the steel ratio.
1. Code suggestion:

\[ a_t = 0.66 + 0.33 \frac{y_1}{x_t} \leq 1.50 \]

- Role of longitudinal steel
  1. It anchors the stirrups, particularly at corners.
  2. It provides dowel resistance.
  3. It controls crack widening.

- Condition of under reinforcement

\[ A_t \leq 2A_t \frac{x_t + y_1}{s} \]

where \( A_t \) = volume per length of longitudinal steel.

\[ \Rightarrow \text{Steel yields first.} \]

- Torsion combined with flexure

- Torsion combined with shear
  - Generally shear exists simultaneously with bending.
    \[ \Rightarrow \text{The existence of shear will reduce the resisting ability in torsion.} \]
    
    Thus, it is necessary to consider the case of torsion combined with shear.
  - For RC beams with transverse reinforcement:
    - Pure torsion: \( T_u = T_c + T_s \)
    - Pure shear: \( V_u = V_c + V_s \)

\[ \frac{T}{T_0} + \frac{2V}{3V_0} = 1 \]

\[ \frac{2T}{3T_0} + \frac{V}{V_0} = 1 \]
ACI Code

- Design for torsion
  - Same interaction as in members without transverse reinforcement.

- Excess torque
  - Over and above that resisted by concrete, the same amount of reinforcement is provided in members subject to torsion plus shear as would be required for purely torsional members.
  - This torsional reinforcement is added to that required for carrying bending moments and flexural shears.

- \( T_u \leq \phi T_n = \phi \left( T_c + T_s \right) \)
  - where \( T_u \) = factored torque,
  - \( \phi \) = capacity reduction factor for torsion = 0.75,
  - \( T_n \) = nominal strength for torsion,
  - \( T_c \) = torsional moment carried by concrete, and
  - \( T_s \) = torsional moment carried by steel.

- \( T_c = \frac{T_0}{\sqrt{1 + \left( \frac{T_0}{V_0} \right)^2 \left( \frac{V_c}{V_T} \right)^2}} \)
  - where \( T_0 = 0.8 \sqrt{f_c x^2 y} \) = pure torsion and \( V_0 = 2 \sqrt{f_c bd} \) = pure shear.
  - \( \frac{T_0}{V_0} = 0.4 \frac{x^2 y}{bd} = 0.4 \frac{C_T}{C_T} \) and \( C_T = \frac{bd}{x^2 y} \)

- Assume \( \frac{V}{T_c} = \frac{V}{T_n} \), such that
\[ T_c = \sqrt{\frac{0.8 \sqrt{f_c x^2 y}}{1 + \left( \frac{0.4 V_u}{C_T \phi T_c} \right)^2}, \quad V_c = \sqrt{\frac{2 \sqrt{f_c bd}}{1 + \left( \frac{2.5 C_T \phi V_u}{V_c} \right)^2}} \]

\[ T_s = \frac{\alpha f_y s}{x_i y_i}, \quad V_s = \frac{A f_y d}{s} \]

\[ T_u = \phi (T_c + T_s) = \phi T_u \]

\[ T_s = \frac{T_u - \phi T_c}{\phi} \]

\[ A_i = \frac{s T_s}{\alpha f_y x_i y_i} = \frac{(T_u - \phi T_c) s}{\alpha f_y x_i y_i} \]

- \( T_s \leq 4T_c \) is required to assure yielding of steel first.
- Minimum spacing of torsional stirrups \( 4(x_i + y_i) \) or 12 in.

- **Condition of neglecting torsional effects**
  - Torsional effects may be neglected if
    \[ T_u < 0.5 \phi \sqrt{f_c \sum_{i=4}^{n} (x_i^2 y_i)} \]
    where \( \sum_{i=4}^{n} (x_i^2 y_i) = \) sum of the small rectangles for irregular shapes.

- **Hollow sections**

![Diagram of a hollow section with dimensions labeled as follows: h for height, y for width, and x for depth.](attachment:diagram.png)
When \( h > \frac{x}{4} \), consider the cross section as solid.

When \( \frac{x}{10} \leq h \leq \frac{x}{4} \), assume it as solid but multiply \( \sum (x^2 y) \) by \( \frac{4h}{x} \).

When \( h < \frac{x}{10} \), consider it as a thin-walled section. \( \Rightarrow \) Check for instability (local buckling).

- General formulation of post-cracking behavior of flexure, shear, and tension interaction in R/C beams

- Discussion of applications: Concrete guideway systems from monorail and maglev transportation infrastructure.

- Design Example – Shear and torsion