Pressure

- Is isotropic - same in all directions
- Is always perpendicular to surface upon which it acts

Hydrostatics

\[ p + \rho gh = \text{constant} = p_0 + \rho gh_0 \]

Valid within fluid of constant density at rest. Constant determined by knowledge of pressure \( p = p_0 \) at reference elevation \( z = z_0 \).

Hydrostatic Forces

- Horizontal force on surface = Horizontal force on projection of surface onto vertical plane
- Vertical force on surface = Weight of fluid "above" surface (or its translation horizontally to a location where there is fluid above)

**Moments**

\[ M = \text{Force} \times \text{Arm} \]
Bemocilli

Along a Streamline, \( s \)

\[
\frac{1}{2} \rho V_s^2 + p_s + \rho g z_s = \text{Constant}
\]

for steady flow \((\frac{\partial t}{\partial t} = 0)\). Constant obtained from knowledge of \( V_s \), \( p_s \) at reference point of streamline \( z = z_s \).

Perpendicular to Streamline, \( n \)

\[
\rho n + \rho g z_n = -\rho \int (V_s^2/R) n \, dq + \text{Constant}
\]

for steady flow with \( n \perp s \) and pointing towards center of curvature of \( s \). \( R \) = radius of curvature of streamline. If \( R \to \infty \) = straight streamlines \( \rho \) pressure varies hydrostatically normal to straight parallel streamlines.

Conservation of Mass

\[
\frac{\partial}{\partial t} \int \rho \, dq = \int \rho (\vec{v} \cdot \vec{n}) \, ds = \int \rho q_n \, ds - \int \rho q_t \, ds_{\text{in}} - \int \rho q_t \, ds_{\text{out}}
\]

If \( \rho = \text{constant} \) over flow areas:

\[
\int q_n \, dA = \rho \int q_t \, dA = \rho Q = \rho \bar{v} A
\]

\( Q = \text{Discharge}, \ \bar{v} = \text{average velocity} = Q/A \)

If fluid incompressible: Volume Conservation

\[
\frac{dV}{dt} = \Sigma Q_{\text{in}} - \Sigma Q_{\text{out}} \quad V = \text{volume of fluid within chosen boundaries or fixed control volume}
\]

Geometry

Area of circle: \( \pi r^2 \); Circumference = \( 2\pi r \)

Volume of sphere: \( \frac{4}{3} \pi r^3 \); Surface area = \( 4\pi r^2 \)
MOMENTUM

\[ Q = VA \quad V = Q/A \]

Well behaved flow: Streamlines straight & parallel \( \perp A \)

\[ \overrightarrow{MP} = (\rho V^2 + P_{ce}) \overrightarrow{A}, \perp A \text{ towards control volume} \]

\( P_{ce} = \) pressure at center of gravity of \( A \)

Equilibrium of forces (steady flow):

\[ \overrightarrow{MP_1} + \overrightarrow{MP_2} + (\text{sum of all other forces on fluid within } V_0) = 0 \]

"Other forces": Shear forces & pressure forces on boundaries of \( V_0 \), gravity, drag forces on objects in \( V_0 \)

DRAG FORCE:

\[ F_D = \frac{1}{2} \rho C_D A_1 V^2 \]

\( A_1 = \) area of body's projection on plane \( \perp V \)

\( C_D = \) drag coefficient \( = C_D(Re) \)

BERNOULLI

\[ H = \text{Total Head} = \frac{V^2}{2g} + \frac{P_{ce}}{\rho g} + Z_{ce} \]

Piezometric Head = \( \frac{P_{ce}}{\rho g} + Z_{ce} \); Velocity Head = \( \frac{V^2}{2g} \)

EGL = Energy Grade Line: \( Z_{EGL} = H \)

HGL = Hydraulic Grade Line: \( Z_{HGL} = H - \frac{V^2}{2g} \)

Flow from 1 to 2 with well behaved flow @ 1 & 2

\[ H_1 = H_2 + \Delta H; \Delta H = \text{head loss between 1 & 2} \]
HEAD LOSSES

$\Delta H = 0$ for short transition with converging flows.

Pipe friction losses

$\Delta H_f = f \frac{L}{D} \frac{V^2}{2g}$  
$D = \frac{A}{\text{Perimeter}} = \frac{4}{\text{Hyd. Radius}}$

$f = f \left( \frac{V_D}{V} , \frac{D}{D} \right)$ from Moody  
$\varepsilon = \text{pipe roughness}$

Wall shear stress: $\tau_w = \frac{f}{8} \rho V^2$

Minor losses

$\Delta H_m = k_2 \frac{V^2}{2g}$  
$k_2 = \text{Minor Loss Coefficient}$

Expansion loss: $\Delta H_{exp} = \frac{(V_2 - V_1)^2}{2g}$

Exit loss ($R_2 > R_1$): $k_{exit} = 1$

Entry loss (sharp edged orifice): $k_{entry} = \left( \frac{1}{C_c} - 1 \right)^2$

$C_c = \text{Contraction Coefficient}$  
$[C_c = 0.6 \rightarrow \frac{1}{1}, C_c = 0.5 \rightarrow \frac{1}{2}]$

ENERGY

$E =$ rate of flow of Mech. Energy  
$\rho g Q (H_1 - H_2)$ = rate of dissipation of Mech. Energy between

1. & 2. $[\text{power loss}]$ = rate of production of internal energy

PUMPS & TURBINES

$\text{BHP} = \eta \rho g Q \left[ H_{out} - H_{in} \right]$  
$\eta =$ efficiency $\leq 1$

For pump, flow of energy is reversed

$\eta \text{BHP} = \rho g Q \left[ H_{in} - H_{out} \right]$  
$[\text{Pumps}]$
**UNIFORM FLOW**

\[
V = \sqrt{\frac{2g}{\rho}} R_h^{\frac{1}{2}} S_o^{\frac{1}{2}} = C R_h^{\frac{1}{2}} S_o^{\frac{1}{2}} = \frac{1}{n} R_h^{2/3} S_o^{1/2} = \frac{Q}{A}
\]

Darcy-Weisbach, Chezy, Manning

- \( S_o \): channel slope
- \( R_h \): hydraulic Radius = \( \frac{A}{P} \)
- \( \frac{Q}{b_s} \): \( \frac{Q}{b_s} = \frac{V^2}{g} \) = \( \frac{V^2}{g A^3} \) = \( \frac{V^2}{g (A/b_s)} \) = \( \frac{V^2}{g h_m} \)

\( S_f \): slope of EGL = \( S_o \): Uniform Flow.

\( n \): Manning's \( n = 0.038 \varepsilon^{1/6} \) (SI-units)

**ENERGY PRINCIPLE**

\[
H = \frac{V^2}{2g} + h + z_o
\]

\( E \): Specific Energy = \( H - z_o \)

\[
E = \frac{Q^2}{2g A^2} + h
\]

**MOMENTUM PRINCIPLE**

\[
\frac{MP}{g A} = \frac{Q^2}{g A} - (h - y_{cc}) A
\]

\( y_{cc} \): y-value of centroid of A

(OVER)
HYDRAULIC JUMP (UNASSISTED) IN RECTANGULAR CHANNEL

$Fr_1 > 1, \quad Fr_2 < 1$

$h_2 = \frac{1}{2} \left( -1 + \sqrt{1 + 8 Fr_1^2} \right), \quad h_1 = \frac{1}{2} \left( -1 + \sqrt{1 + 8 Fr_2^2} \right)$

$\Delta H_{jump} = H_1 - H_2 = E_1 - E_2 = \left( \frac{V_1^2}{2g} + h_1 \right) - \left( \frac{V_2^2}{2g} + h_2 \right) = \frac{(h_2 - h_1)}{4h_1 h_2^3}$

GRADUALLY VARIED FLOW

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

$S_o = S_f$ for uniform flow = Normal Depth

$S_f$ replaces $S_o$ in formulas for $V$

$Fr^2 = 1 = \text{Critical Depth}$

$S_f = \frac{Q^2}{8g A^3/P} = \frac{C^2}{A^3/P} = \frac{n^2 Q^2}{A^{1.3}/P^{0.5}} = \frac{T_s}{g A/P}$

Darcy-Weisbach Chezy Manning

GRADUALLY VARIED FLOW PROFILES

Mild Slope

Steep Slope

\[ \text{Diagram of gradually varied flow profiles with mild and steep slopes.} \]