General Comment

The test is concerned with various aspects of a single problem, and consists of questions requiring you to have solved previous questions. For this reason "default" answers are given so that you can proceed. The "default" answers are not necessarily the correct solutions (but they may be close) so continue to use your own solutions unless they differ from the default values by more than 10%. In answering some of the questions you may find the figure below helpful.

GOOD LUCK!

Generalized Moody Diagram: $D = 4R_h = 4(A/P)$
General Problem Description

A 10-m-wide concrete caisson is being transported down a river by tug boats when the mooring lines break. The caisson floats freely down the river and comes to rest against the bank abutments of a bridge thereby creating an obstruction to the natural flow in the river. After a transitional period, during which the river flow adjusts to the presence of the caisson, steady flow conditions are achieved. The sketch below (not to scale) shows the steady state flow scenario in the vicinity of the flow obstruction formed by the caisson.

The caisson spans the entire width of the river and leaves a gap of uniform height \( h_g = 0.4 \) m, length \( b = \) width of caisson \( = 10 \) m, and width (in direction into the paper) of \( B = \) width of the river \( = 50 \) m between the caisson and the river bottom. Over the distance covered by the sketch the river bottom may be assumed horizontal at \( z = 0 \). Thus, the discharge in the river, \( Q = 100 \) m\(^3\)/s, must pass under the caisson, i.e. through a closed conduit of "very" rectangular cross-section, \( A_x = h_x B \), and length \( b = 10 \) m.

Due to the flow obstruction created by the caisson the water (\( \rho = 1,000 \) kg/m\(^3\) and \( v = 10^{-6} \) m\(^2\)/s) is backed up to a depth \( h_1 \) a short distance (~20 m) upstream of the caisson. The flow enters the gap between the caisson and the river bottom through a contraction (caisson has sharp corners and the contraction coefficient is \( C_r = 0.6 \)), and shoots out from under the caisson at section 2-2, where the depth, measured along the downstream sidewall of the caisson, is \( h_2 \). Following exit from under the caisson the flow expands and forms a uniform flow of depth \( h_3 = 3.0 \) m, a relatively short distance (~20 m) downstream of the caisson.
Question No: 1 (10%)

Determine the velocity in the gap below the caisson, \( V_s \), and show that the velocity heads of the flows upstream and downstream of the caisson where the depths are \( h_1 > h_3 = 3.0 \, \text{m} \), are negligibly small compared to the velocity head of the flow in the gap between caisson and river bottom (Default values \( V_s = 5 \, \text{m/s} \), \( V_1 = 0.7 \, \text{m/s} \)).

Question No: 2 (22%)

Consider the flow expansion that takes place after the flow exits from under the caisson (at 2-2) to the uniform river flow achieved a short distance (~20 m) downstream of the caisson, where the depth is \( h_1 = 3.0 \, \text{m} \), and determine the depth \( h_2 \) along the downstream sidewall of the caisson. (Default value: \( h_2 = 2.7 \, \text{m} \)).

Question No: 3 (33%)

Determine the depth \( h_1 \) a short distance (~20 m) upstream of the caisson required to drive the discharge \( Q \) under the caisson. Assume (i) the depth at the outflow from under the caisson to be \( h_2 = 2.7 \, \text{m} \) (regardless of the value you obtained in Q #2); (ii) the caisson and the river bottoms to have the same roughness, \( e = 0.5 \, \text{cm} \); (iii) the caisson to have sharp corners so that \( C_r = 0.6 \); and (iv) the velocity head at 1-1 is negligibly small, as was shown in Q #1). (Default value \( h_1 = 5 \, \text{m} \)).

Question No: 4) (10%)

Determine the total headloss, \( \Delta H_{1-3} \), caused by the flow obstruction created by the caisson, and the portion of this headloss contributed by the flow expansion from 2-2 to 3-3, \( \Delta H_{2-3} \). (Default values \( \Delta H_{1-3} = 2 \, \text{m} \), \( \Delta H_{2-3} = 1 \, \text{m} \)).

Question No: 5 (15%)

Sketch the EGL and HGL for the flow in the closed conduit of length \( b = 10 \, \text{m} \) formed by the gap between the caisson and the river bottom.

Question No: 6 (10%)

Determine the total horizontal force acting on the fluid from the surrounding boundaries between 1-1 and 3-3. Is this horizontal force equal to the horizontal force acting from the fluid on the caisson?
Solutions

Question No: 1
Steady flow ⇒ Conservation of volume ⇒ Q = V/R = Const.
Q = 100 m³/s; \( A_2 = h_2 \cdot B = 0.4 \cdot 50 = 20 \text{ m}^2 \); \( V_2 = Q/A_2 = 5 \text{ m/s} \)
\( V_3 = Q/A_3 = Q/(h_3 \cdot B) = 100/(3.50) = 0.67 \text{ m/s} \)
\( V_1 = Q/A_1 < V_3 \) since \( A_1 = h_1 \cdot B > A_3 = h_3 \cdot B \).
\( V_3^2/2g = \text{vel. head} @ 3 = (0.67)^2/2.98 = 0.02 \text{ m} \) \( \ll V_2^2/2g = 1.28 \text{ m} \);
\( V_1^2/2g < V_3^2/2g \approx 1.5\% \) of \( V_2^2/2g \): Negligible

Question No: 2
Steady flow ⇒ short transition (no friction force/loss)⇒ expanding flow (expansion loss \( \Delta H \)) ⇒ Momentum & Volume conservation must be used! ! between 2-2 and 3-3:
\[
\frac{1}{2} \rho g h_2^2 + \frac{1}{2} \rho V_2^2 + \frac{1}{2} \rho V_3^2 = \frac{1}{2} \rho g h_3^2 + \frac{1}{2} \rho V_3^2
\]
\[
h_2 = \sqrt{h_3^2 - 2(Q/B)(V_3 - V_2)V_2}
\]
\[
\sqrt{3^2 - \frac{1}{2} \left(\frac{100}{50}\right) (5.0 - 0.67) V_2^2} = \sqrt{9 - (2.04 - 0.27)} = 2.69 \text{ m}
\]
If \( \rho V_3^2 \) (\( M_3 \)) is neglected: \( h_2 = 2.64 \text{ m} \)

Note: Approximate same answer if
\[
H_2 = h_2 + V_3^2/2g = H_3 + \Delta H_3 = h_3 + V_3^2/2g + \frac{(V_3 - V_2)^2}{2g}
\]
Question No: 3

There will be pressure forces on fluid between 1-1 and 2-2 from the upstream sidewall of the caisson. We can not assume hydrostatic pressure as we don't know forces acting on fluid. Bernoulli and volume conservation must be used between H 1-1 and H 2-2.

\[ H_1 = H_2 + \Delta H_{1-2} = H_2 + \Delta H_f + \Delta H_m \]

\[ H_1 = \frac{V_i^2}{2g} + \frac{P_{cs,1}}{g} + Z_{cs,1} = \frac{V_i^2}{2g} + \frac{P_{cs,1} + \rho g z_{cs,1}}{g} \]

but \( p + \rho g Z = \text{constant at 1-1} \) (well-behaved flow)\( \Rightarrow \)

\[ H_1 = h_1 + \frac{V_i^2}{2g} \quad (V_i^2/2g \text{ negligible from Q #1, so it can be dropped}) \]

\[ H_2 = \frac{V_q^2}{2g} + \frac{P_{cs,2}}{g} + Z_{cs,2} = \frac{V_q^2}{2g} + h_2 \]

\( Z_{cs,2} = h_2/2 = 0.2 \text{ m} \); \( P_{cs,2} = \text{pressure in receiving fluid} = \rho g (h_2 - Z_{cs,2}) \)

\[ \Delta H_f = f\left(\frac{L}{4R_h}\right) \frac{V_q^2}{2g} = f \frac{b}{4R_h} \frac{V_q^2}{2g} \quad (b = 10 \text{ m}) \]

\[ R_h = \frac{A_q}{\rho g} = \frac{h_g \cdot B}{28 + 2h_g} = \frac{0.4 \cdot 50}{2.00 + 2.64} = 0.2 \text{ m} \left(= \frac{h_g}{2} \right) \]

\( 4R_h = D = 2h_g = 0.8 \text{ m} \)

\[ Re_q = \frac{V_q (4R_h)}{\nu} = \frac{5.0 \cdot 0.8}{10^{-6}} = 4.10^6 \quad ; \quad 4R_h = \frac{0.5 \cdot 10^{-2}}{0.8 \text{ m}} = 6.2 \cdot 10^{-3} \]

Moody Diagram: \( f = 0.032 \) (was 0.02!)
\[ \Delta H_p = f \cdot \frac{b}{4R_n} \frac{V_p^2}{2g} = 0.032 \cdot \frac{10}{2g} \frac{V_p^2}{2g} = 0.4 \frac{V_p^2}{2g} \]

\[ \Delta H_m = \frac{V_m}{2g} = \left( \frac{1}{C_v} - 1 \right)^2 \frac{V_p^2}{2g} = 0.44 \frac{V_p^2}{2g} \]

\[ h_i = h_2 + \left(1 + 0.4 + 0.44 \right) \frac{V_p^2}{2g} - \frac{V_i^2}{2g} = h_2 + 1.84 \frac{V_p^2}{2g} - \frac{V_i^2}{2g} \]

\[ v_i^2/2g < 1.84 V_p^2/2g \quad \text{(from Q #1) so drop it} \]

\[ \frac{V_i^2}{2g} = \left( \frac{100}{50.525} \right)^2 \frac{2g}{2g} < 0.01 \text{ m}^3 \]

\[ \text{(Check: } V_i^2/2g = \left( Q/8h_i \right)^2/2g = \left( \frac{100}{50.525} \right)^2/2g < 0.01 \text{ m}^3 \)

**Question No: 4**

Headloss: Obviously Bernoulli is to be used

\[ H_1 = h_1 + \frac{V_1^2}{2g} = H_3 + \Delta H_{i-3} + \frac{V_3^2}{2g} + \Delta H_{i-3} \]

\[ \Delta H_{i-3} = h_i - h_3 - \frac{V_3^2 - V_i^2}{2g} = 5.05 - 3.0 - \left( 0.02 - 0.01 \right) = 2.05 m \]

Bernoulli from 2-2 to 3-3 gives

\[ H_2 = h_2 + \frac{V_2^2}{2g} = H_3 + \Delta H_{2-3} = h_3 + \frac{V_3^2}{2g} + \Delta H_{2-3} \]

\[ \Delta H_{2-3} = h_2 - h_3 + \frac{V_2^2}{2g} - \frac{V_3^2}{2g} = 2.7 - 3.0 + \frac{5^2}{2g} \]

\[ = 0.98 m \quad \text{from Q #1} \]

[Note: If using simple formula for expansion loss]

\[ \Delta H_{exp} = \left( V_3 - V_1 \right)^2/2g = (5 - 0.67)^2/2g = 0.96 m \]

It would work here, BUT THIS IS NOT ALWAYS SO!]
Question No: 5

$$V_1^2/2g = 0.006 \text{ m} = 0; \quad V_g^2/2g = 2.98 = 1.28 \text{ m}$$

$$\Delta H_m = 0.44 V_g^2/2g; \quad V_{v.c.}/V_g = 1/0.6 = 1.67$$

$$V_{v.c.}^2/2g = 2.8 \cdot V_g^2/2g$$

Start

Both start at \(2 = h_1\) (since \(V_1^2/2g = 0\))

\(Z_{EGL}\) unchanged up to Vena Contracta (of converging flow)

\(Z_{HGL}\) drops to a distance of \(V_{v.c.}^2/2g = 2.8 V_g^2/2g\) below

\(Z_{EGL}\) at Vena Contracta (\(V\) increases, \(p\) decreases)

Expansion after Vena Contracta:

\(Z_{EGL}\) drops by \(\Delta H_m = 0.44 V_g^2/2g\)

\(Z_{HGL}\) rises from low point to be \(V_g^2/2g\) below \(Z_{EGL}\)

Rest of the way:

\(Z_{EGL}\) varies linearly until end, where it is \(V_g^2/2g\) above \(2 = h_2\). Decrease is due to \(\Delta H_f\)

\(Z_{HGL}\) varies linearly - is parallel to EGL since \(V = V_g\) - constant - and meets the free surface at exit from under caisson.
**Question No: 6**

**Momentum between 1-1 and 3-3**

\[ MP_1 = MP_3 + F_h \]

\[ F_h = \text{horizontal force on fluid} \]

\[ F_h \text{ (positive in upstream)} = MP_1 - MP_3 = (\frac{1}{2} \rho g h_1^2 + \rho V_1^2 h_1)B - (\frac{1}{2} \rho g h_3^2 + \rho V_3^2 h_3)B = (125 \times 10^3 + 0.8 \times 10^3)B - (44 \times 10^3 + 1.33 \times 10^3)B \]

**Note:** Low contributions of \( \rho V^2 \) terms to MPS

**Reason:** Smallness of \( V^2/2g \) & \( V^2/g \) shown in Q-1

\[ F_h = 4.02 \times 10^6 N \]

But part of this force comes from the shear stress acting on the river bed under the caisson. With \( T_s = \frac{1}{6} f_0 V_g^2 = \frac{1}{6} \times 0.032 \times 1000 \times 5^2 = 100 \text{ N/m} \)

This force is \( F_\text{KB} = T_s \cdot b \cdot B = 100 \times 10 \times 50 = 5 \times 10^6 \text{ N} \)

i.e. about \( \approx 1\% \) of total force - could be neglected. For completeness:

\[ F_c = \text{Force on Caisson} = F_h - T_s \cdot B = 3.97 \times 10^6 \text{ N} = 4 \text{ MN} \]

**acting in the downstream direction.**