LECTURE #1

1.060 ENGINEERING MECHANICS II

1.050 EMI = Solid Mechanics
1.060 EMII = Fluid Mechanics

Common: Mechanics, i.e. based on Newton's Laws & Continuum Hypothesis
Difference: Material whose behavior we want to describe.

Definition of a FLUID:

A fluid is a substance that cannot support a shearing force (shear stress) without being in motion.

or, in the vocabulary of 1.050,

A fluid is a *Tresca Material* with zero cohesion, i.e. unless \( \sigma_1 = \sigma_2 = \sigma_3 \)
the fluid is in a state of failure.

There are two types of fluids:
- Liquids (exhibit a "free surface")
- Gasses (need a "lid" to be contained)
but same principles (often) apply to both.
In **solid mechanics** we need to determine stresses, $\sigma_{ij}$, and displacements, $\xi_i$, from external forces, material properties and boundary conditions.

In **fluid mechanics** we need to determine stresses, $\sigma_{ij}$, and rates of displacements, $\dot{\xi}_i = \text{velocities} = \mathbf{u}_i$, from external forces, material properties and boundary conditions.

**Stress tensor in solid mechanics**

$$\sigma_{ij}$$

is replaced by

$$\sigma_{ij} = -p \delta_{ij} + \tau_{ij}$$

where

\[ p = \text{fluid pressure (positive for compression)} \]

\[ \delta_{ij} = \text{Kronecker Delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \tau_{ij} = \text{viscous stress tensor} \]

\( \text{Same sign convention} \)

\( \tau_{ij} \) as in solid mech., i.e. $\tau_{xx}$, $\tau_{yy}$, $\tau_{zz} > 0$ for tension

in fluid mechanics.

**Fluid pressure is isotropic** (same in all direction) and it follows from the definition of a fluid that $\tau_{ij} = 0$ if the fluid is at rest or in motion as a solid body.
The nature of viscous (shear) stresses in a moving fluid is illustrated in the sketch below:

Layer 1 moves at velocity \( \sim U_{S1} \)
Layer 2 moves at velocity \( \sim U_{S2} \)

Relative to layer 2, the fluid in layer 1 moves faster, i.e., it slides over 2, and tries to "pull" the fluid in layer 2 along. Similarly, the fluid in layer 2 tries to "hold back" the fluid in layer 1.

This interaction is equivalent to a shear stress acting at the common boundary between 1 and 2:

\[ \tau_{ns} = \mu \frac{\partial U_s}{\partial n} = \rho \nu \frac{\partial U_s}{\partial n} \]

\( \mu \) = dynamic viscosity
\( \rho \) = fluid density
\( \nu \) = kinematic viscosity \( (\mu/\rho) \)

If there is no "shear" in the velocity field of a fluid, i.e., \( \frac{\partial U_i}{\partial x_j} = 0 \), then there can be no viscous stresses, i.e., \( \tau_{ij} = 0 \).
The use of differential calculus, e.g. the term $dU/dn$ in the expression for the viscous shear stress, in fluid (as well as in solid) mechanics is based on the Continuum Hypothesis.

As an example illustrating fluid, we take Fluid density at a point = $\rho(x, y, z)$

By definition

$$\rho(x, y, z) = \lim_{\Delta V \to 0} \left( \frac{\Delta m}{\Delta V} = \frac{N m_v}{\Delta V} \right)$$

$N$ molecules of mass $m_v$ in $\Delta V$

If $\Delta V \to 0$ we may "hit" a molecule or we may not. Even if we "hit" a molecule, it may be gone in an instant.

With $\Delta V \to 0$ = "absolute zero" $\rho$ is undefined!

With $\Delta V \to 0$ = a volume much smaller in scale than anything we aim to resolve, then $\rho$ at a "point" is defined.

$N = 10^7$ (liquid); $10^8$ (gas) if $\Delta V = 1 \mu m^3$

A cube of side length = $1 \mu m = 10^{-3} \text{ mm} = 10^{-6} \text{ m}$

For $\Delta V = 0 = O(\mu m^3)$ $N$ may not be exactly constant, but even if it varied $\pm 100$ that would only be $10^{-7}$%, i.e., nothing!