Uniform pipe ⇒ A = constant, P = constant
Discharge = Q = constant ⇒ V = QA = constant

\[ H_1 - H_2 = \left( \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + z_1 \right) - \left( \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + z_2 \right) = \]
\[ \left( \frac{P_1}{\rho g} + z_1 \right) - \left( \frac{P_2}{\rho g} + z_2 \right) = \text{difference in piezometric head between 1 and 2} \]
what is obtained from a manometer connected to the pipe at 1 and 2 [Lecture #4]

\[ \Delta H_f = \int_{z_1}^{z_2} \frac{T_3 P}{\rho g A} \, ds = \frac{T_3 P}{\rho g A} l_{12} \] (since conditions are uniform along the pipe)
**Dimensional Analysis**

What are we looking for?

Dependent Variable: \( \tau_s = \) wall shear stress

What can we change?

Independent Variables:

Fluid: \( \rho \) and \( \nu = \mu / \rho \)
Pipe dimension: \( D = \) diameter \( [\text{we choose } 0] \)
Fluid Velocity: \( V \) \( [\text{note: } Q = VA \text{ is known}] \)
Pipe wall roughness: \( \varepsilon \)

Basic Dimensions (units)

\[ \begin{align*}
\text{led} &= D \\
\text{ted} &= D / V \\
\text{med} &= \rho D^3
\end{align*} \]

Independent Variables Left:

\[ V = [V] = \text{led}^2 / \text{ted} = D^2 / (DV) = DV \]

\[ \Pi_1 = \frac{V}{DV} \quad [\text{non-dim. viscosity}] \]

\[ \varepsilon = [\varepsilon] = \text{ted} = D \]

\[ \Pi_2 = \frac{\varepsilon}{D} \quad [\text{non-dim. wall roughness}] \]

Dependent Variable

\[ \tau_s = [\tau_s] = \text{force/area} = \text{med} (\text{led} / \text{ted}^2) / (\text{led})^2 = \rho D^3 (D / (DV)^2) / D^2 = \rho V^2 \]

\[ \Pi_D = \frac{\tau_s}{\rho V^2} \quad [\text{non-dim. wall shear stress}] \]
Result of Dimensional Analysis

\[ \frac{\bar{w}}{D} = \frac{T_s}{\rho V^2} = f_0 \left( \frac{v}{D} = \text{Re} = \text{Reynolds Number} \right) \]

or with

\[ \bar{w} = \frac{D V}{v} = \text{Re} = \text{Reynolds Number} \]

and

\[ \bar{T}_s = \frac{\varepsilon}{D} = \text{Relative Roughness} \]

\[ T_s = \frac{1}{8} f(\text{Re}, \varepsilon/D) \rho V^2 = \frac{1}{8} f_0 \rho V^2 \]

where

\[ f = 8 f_D = f \left( \text{Reynolds Number} = \frac{D V}{v}, \text{Rel. Rough.} = \frac{\varepsilon}{D} \right) \]

The Darcy-Weisbach Friction Factor

If pipe is smooth \( \varepsilon = 0 \)

\[ f = f \left( \text{Re} = \frac{D V}{v} \right) \text{ for smooth walls} \]

Reynolds Experiment

Smooth circular pipe

\[ \text{Re < Re_{crit}} \quad \text{Re_{crit} \approx 2 \cdot 10^3} \quad \text{Re > Re_{crit}} \]

Laminar Flow \quad Turbulent Flow
103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However, the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.
Transition from laminar to turbulent flow when
\[ Re \approx Re_{\text{crit}} \approx 2 \times 10^3 \]
For water:
\[ Re = \frac{DV}{\nu} = \frac{DV}{\frac{10^{-6} \text{m}^2}{\text{s}}} = 2 \times 10^3 = Re_{\text{crit}} \]
\[ (D,V)_{\text{crit}} \approx 2 \times 10^{-3} \text{ m}^2/\text{s} \]

If \( D = 1 \text{ cm} = 0.01 \text{ m} \) [quite small]
Flow is turbulent if \( V > \sim 0.2 \text{ m/s} \) [very low - it would take approximately 1 min
to fill a 1 liter bottle at this flow rate,
\[ Q = V \cdot (\pi/4) D^2 = 0.016 \text{ liters/s} \]

For our fluid of choice - WATER -
flows of interest to us are TURBULENT.

Thus, in principle all flows of interest
to us are inherently UNSTEADY due to
the chaotic nature of turbulent flows.
We remove this obstacle by expanding
our definition of what we consider
to be "small", i.e. of a spatial or
temporal scale below the scales of
interest to us, by averaging over a
a time interval \( \delta t \) larger than the
time scale of turbulence, but still smaller
than the temporal resolution of the flow we seek to resolve by our analysis.

\[ U(t) = U + U' \]
\[ U = \frac{1}{\delta t} \int u(t) \, dt \]
\[ \int \delta t \, dt = 0 \]

If \( U \) does not change over times much much much larger than \( \delta t \) we consider the flow steady.

If \( U \) changes over times somewhat larger than \( \delta t \), then we consider the flow unsteady.

If mean value \( U(t) = \int_{t_0}^{t} u(t) \, dt \) is not a function of \( t_0 \) flow is steady, and if \( U(t_0) \) varies with \( t_0 \) the flow is unsteady.