Analysis of Pipe Flow

Fluid: Water
\( \rho = 10^3 \text{ kg/m}^3 \)
\( \mu = 10^{-6} \text{ m}^2/\text{s} \)

260 m long, cast iron pipe, \( D = 0.26 \text{ m} \)

Head in supply tank = \( H_i = h_1 \)
Head in receiving tank = \( H_{rec} = h_2 \)

Total headloss in system = \( \Delta H = h_1 - h_2 \)

\( \Delta H = \sum \Delta H_f + \sum \Delta H_{\text{minor}} \)

Here: \( \Delta H_f = f \frac{L}{D} \frac{V^2}{2g} \) (only one pipe)

\( H_i = h_i \)
\( H = \frac{P}{\rho g} + z + \frac{V^2}{2g} \)
\( V = \frac{Q}{A} \)

\( \Delta H_{in} = K_{\text{in}} \frac{V^2}{2g} \)

\( \Delta H_{out} = \frac{V^2}{2g} \)

\( H_{rec} = h_2 \)
Choice "End" point

\[ p_2 + \rho g z_2 = \rho g h_2 \]
\[ v^2/2g = \text{vel. head} \]
\[ H_2 = h_2 + \frac{v^2}{2g} \]

Difference is (of course) the velocity head \( \frac{v^2}{2g} \) which is the head loss of the outflow into a large reservoir!

But if outflow is free then what?

\[ p = 0, \quad z = z_2, \quad \frac{v^2}{2g} \]
\[ H_{\text{end}} = z_2 + \frac{v^2}{2g} \quad \text{Correct} \]

\[ \rho \rho g z = \rho g h_2 \]
\[ H_{\text{end}} = h_2 \quad \text{Incorrect} \]

Difference = \((z_2 - h_2) + \frac{v^2}{2g}\) is NOT just the velocity head! Reason: Impact of free jet into pool causes dissipation.

Proper Choice: Take "end" at exit from pipe

If \( H_{\text{end}} = h_2 \) lowering pool level would increase \( h_1, h_2 \) and increase flow which is ABSURD!
\[ H_i = H_i^0 + \Delta H = h_2 + \frac{V^2}{2g} + \frac{V^2}{2g} \frac{f_L l}{D} \]

\[ h_1 - h_2 = (\Sigma K_L + f \frac{L}{D}) \frac{V^2}{2g} \]

\[ V = \left\{ \frac{2g (h_1 - h_2)}{(\Sigma K_L + f \frac{L}{D})} \right\}^{1/2} \quad (1) \]

Assume rounded inlet conditions: \( \Sigma K_L = K_L = L = 1 \)

\( h_1 - h_2 = 1 \text{ m. BUT WE DON'T KNOW } f! \)

\[ f = f \left( Re = \frac{VD}{\nu} , \frac{\varepsilon}{D} \right) : \text{MOODY} \quad (2) \]

\( \varepsilon = 0.26 \text{ mm, } 10^{-3} \)

BUT WE DON'T KNOW \( V \) (until we know \( f \)!

**Method 1:** Take "standard" \( f = f^2 = 0.02 \),
get \( V = V^{(2)} \) from (1) then \( f = f^{(3)} \) from Moody with \( Re = Re^{(2)} = V^{(2)} D/\nu \). Go back to (1) etc until \( V^{(n+1)} \sim V^{(n)} \)

**Method 2:** Assume Fully Rough Turbulent Flow
and get \( f = f^{(2)} = f(Re \to \infty, \varepsilon/D) \) from Moody,
then get \( V = V^{(2)} \) from (1), now \( Re = Re^{(2)} = V^{(2)} D/\nu \) & Moody gives \( f = f^{(3)} \) etc.
Computations for \( h_1 - h_2 = 1 \text{m} \)

Cast Iron : Table 8.1 \( \varepsilon = 0.26 \text{mm} \)

Relative Roughness = \( \varepsilon / D = 0.26 \text{mm} / 0.26 \text{m} = 10^{-3} \)

If flow is R.T. = Moody gives \( f'' = 0.0196 \)

[very close to "standard" value of 0.02 used in Method 1 - here the two approaches are the same]

\[
V = \frac{\sqrt{2g(h_1 - h_2)}}{\sqrt{1 + f''(\varepsilon/D)}} = \frac{\sqrt{2 \times 9.8 \times 1}}{\sqrt{1 + 0.0196 \times \frac{0.26}{0.26}}} = \frac{\sqrt{19.6}}{\sqrt{1 + 1.96}} = 0.98 \text{ m/s}
\]

Minor loss is minor

Now \( \text{Re}'' = V''D / \nu = 0.98 \times 0.26 / 10^{-6} = 2.5 \times 10^5 \)

and \( \varepsilon / D = 10^{-3} \) gives

\( f'' = 0.0209 \)

\[
V = \frac{\sqrt{2g(h_1 - h_2)}}{\sqrt{1 + f''(\varepsilon/D)}} = \frac{\sqrt{19.6}}{\sqrt{1 + 20.9}} = 0.95 \text{ m/s}
\]

Now \( \text{Re}'' = V''D / \nu = 2.47 \times 10^5 \approx 2.5 \times 10^5 \) same as before. Therefore same \( f'' = f'' \) and we're done!

\[
V = V'' = 0.95 \text{ m/s} \Rightarrow Q = V \cdot A = \frac{\pi}{4} \cdot 0.05 \text{m}^3 / \text{s}
\]

If \( h_1 - h_4 = 4 \text{m} \) not 1m - quick estimate from above \( V \propto \sqrt{h_1 - h_2} \) assuming \( f = f'' = 0.02 \)

\[
V_4 = \sqrt{\frac{4}{1}} V_1 = 1.9 \text{ m/s} \Rightarrow Q = 0.1 \text{ m}^3 / \text{s}
\]

\( \text{Re} = 4.9 \times 10^5 \), \( \varepsilon / D = 10^{-3} \Rightarrow f = 0.0203 \approx 0.0209 \checkmark \)

\( V_4 \) is correct.
EGL: ENERGY GRADE LINE (TOTAL HEAD LINE) &
HGL: HYDRAULIC GRADE LINE (GEOMETRIC HEADLINE)

\[
EGL \quad Z_{EGL} = H = \frac{P_{ce}}{\rho g} + Z_{cg} + \frac{V_i^2}{2g}
\]

\[
HGL \quad Z_{HGL} = \frac{P_{ce}}{\rho g} + Z_{cg} = H - \frac{V_i^2}{2g}
\]

\[
\Delta H_{in} = K_{ce} \frac{V_i^2}{2g} \approx 0.5 \frac{V_i^2}{2g}
\]

\[
\frac{V_i^2}{2g} = \Delta H_{out} \approx \frac{P}{\rho g}
\]

Note: EGL always going down in direction of flow. Not so for HGL

\[
i = V_{ce} \approx 1.6V
\]

\[
\frac{V_i^2}{2g} = 2.6 \frac{V_i^2}{2g}
\]