**LECTURE # 27**

**1.060 ENGINEERING MECHANICS II**

**THE MOMENTUM PRINCIPLE FOR OPEN CHANNELS**

For well-behaved flow—just as in closed conduits—we have in the direction parallel to the bottom (the x-direction)

\[ MP_x = MP = (pV^2 + P_{cg})A = \rho \frac{Q^2}{A} + \rho g (h-y_{cg})A \]

\[ \frac{MP}{\rho g} = \frac{Q^2}{\rho g A} + (h-y_{cg})A \]

For given \( Q \) and Cross-section:

\[ \frac{MP}{\rho g} \rightarrow \infty \text{ as } h \rightarrow 0 : \frac{Q^2}{\rho g A} \rightarrow \infty \]

\[ \frac{MP}{\rho g} \rightarrow \infty \text{ as } h \rightarrow \infty : (h-y_{cg})A \rightarrow \infty \]

Thus, just as we found for the specific energy, \( E \), the Thrust \( \frac{MP}{\rho g} \) must have a minimum value for given \( Q \) and channel cross-section. This minimum is obtained from

\[ \frac{d(MP/\rho g)}{dh} = 0 \]
\[
\frac{\partial (MP/\rho g)}{\partial h} = \frac{\partial}{\partial h} \left[ \frac{Q^2}{gA^2} + hA - y_c A \right] =
\]
\[- \frac{Q^2}{gA^2} \frac{\partial h}{\partial h} + A + h \left( \frac{\partial A}{\partial h} \right) - \frac{\partial (y_c A)}{\partial h} \]
\[
\text{ycA} = \text{moment of A around } y = 0 \quad \delta(y_c A) = (\delta h \cdot b_s) \cdot h \quad \frac{\partial A}{\partial h} = b_s h \quad h \frac{\partial h}{\partial h} = b_s h
\]

Thus,
\[
\frac{\partial}{\partial h} \left( \frac{MP}{\rho g} \right) = - \frac{Q^2 b_s}{gA^2} + \delta y = 0 \quad \text{or}
\]
\[
\frac{Q^2 b_s c}{g A_c^3} = \delta y = 1
\]

How about that! MP is minimum for a given channel carrying a certain discharge Q when the Froude Number is unity, i.e., when the flow is critical, just as we obtained for the minimum of specific energy, E.

The corresponding minimum value is given by
\[
\left( \frac{MP}{\rho g} \right)_{\text{min}} = \left[ \frac{Q^2}{gA_c^2} + (h_c - y_c c) \right] A_c = \left( \frac{A_c}{b_{sc}} + (h_c - y_c c) \right) A_c =
\]
\[
(h_{mc} + h_c - y_c c) A_c
\]

which, for a rectangular channel (\( h_{mc} = h_c \), \( y_c c = \frac{1}{3} h_c \), \( A_c = bh_c \)) is \([MP/b]/(\rho g)_{\text{min}} = (3/2) h_c^2\).
Thus, analogous to the E vs h diagram we have

If $MP > MP_{\text{min}}$, there are two solutions for the depth. One corresponds to supercritical flow, the other to subcritical flow. These depths that correspond to the same value of $MP$ (for given channel geometry and discharge $Q$) are CONJUGATE DEPTHS.

If $MP = MP_{\text{min}}$, there is only one solution corresponding to CRITICAL DEPTH.

If $MP < MP_{\text{min}}$, there is no solution. The specified combination of $MP$ and $Q$ is physically impossible in the given channel.
The Unassisted (Free) Hydraulic Jump

\[ MP_1 + \text{Gravity in } x = MP_2 + \text{Friction forces on bottom} + \text{Other Forces in } x \]

1) Channel must be prismatic and have a plane bottom. If this were not the case, there would be pressure forces from the sidewalls and the bottom that would not be \( 1 \times \), i.e. there would be a contribution of unknown magnitude from "Other Forces in \( x \)."

2) If the plane bottom slope \( \beta \) is small the gravity component \( g_x = g \sin \beta \) would be small. Furthermore, if channel is sloping in the direction of the flow, the friction force from the bottom would counteract the gravity force, so that the net result would be smaller than either of the two contributions.

3) The transition from 1 to 2 is "short". The result is that

\[ MP_1 = MP_2 \Rightarrow h_1 = h_2 \]

which is known as the condition for a Free (or Unassisted) Hydraulic Jump across which the flow changes from supercritical \( (h_1) \) to subcritical \( (h_2) \) [or vice versa - but this turns out to violate energy conservation].
The Assisted Hydraulic Jump

Keeping the channel prismatic of negligible slope and friction (gravity + friction ≈ 0), there may be cases, e.g. in the Shilling Basin down-stream of a dam, when the balance of \( MP_1 = MP_2 \) can not be achieved without some "assistance".

An example of this assistance is baffles, i.e. flow obstructions that produce a drag force on the fluid, that helps to achieve the necessary force equilibrium for the jump:

\[
MP_1 = MP_2 + F_D
\]

Since \( F_D \) would vary depending on the position of the baffles within the jump [large approach velocity = large \( F_D \) if close to start of jump, low velocity = small \( F_D \) if towards end of jump] the baffles can, within a range, adjust \( F_D \) so that the jump takes place across the baffles, i.e. we may control the location of the hydraulic jump, for a range of flow conditions.