Our objective is to measure the Elastic Modulus of steel. The experiment comes in two parts.

In the first, you will subject a steel rod specimen to a tension test using an Instron testing machine designed specifically for that purpose. An "extensometer" will be used to obtain a measure of strain and a "load cell" used to obtain a measure of stress.

In the second part of the experiment, you will do a tension test of a stainless steel wire, loading the specimen using the dead weights available in the lab but measuring displacement using the LVDT, Linearly Variable Differential Transformer.

You will use the pages that follow in reporting your results. You will fill in the blanks with the data you collect, with the results of the calculations specified herein, draw or paste in graphs in the spaces provided, and provide an analysis of your results when requested. This then is your report.

2.1 Tension Test of a steel rod.

The INSTRON machine is built specifically to subject structural elements to tension or compression. The schematic at the right indicates how it works.

A double acting piston drives the table up, or down, when pressurized hydraulically. The test specimen is fixed relative to the top bar and the shaded cross bar by the grips; the shaded cross bar is fixed in space. Hence, the specimen is subject to a tensile load as the table moves downward.

Test Sequence - Overview¹

Measure and record the diameter of the test specimen.

Fix the specimen in the grips of the machine.

Attach the extensometer, recording the gage length (below)

Set up the Instron for "ramp" increase of load, setting the actuator control for a table motion of 0.2in / min.

Do "automatic calibration" to set full scale values for load and displacement and output voltages. Record load and displacement factors. (below)

Ensure the Data Acquisition System is set to record.

Load specimen to rupture. Wear safety glasses.

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1. Steve Rudolph and Pong will set up and run the test machine. You are responsible only for the first step.
Parameters for determining tensile stress.

The load is sensed by a load cell at the top bar. It puts out a voltage proportional to the load. For our purposes, all you need know is the scaling factor for converting the voltage to tensile force. This is obtained from a “full scale” setting, the “automatic calibration” of the load cell. At full scale, the signal output is 4.0 Volts and the corresponding load is 100 KN or ___________ Kips1.

The load factor is then:

\[
\text{Load factor } = \frac{\text{KN}}{\text{volt}} \quad \text{or} \quad = \frac{\text{Kips}}{\text{volt}}
\]

Knowing the cross-sectional area of the test specimen, we can, knowing the load, compute the tensile stress in the bar. We record that the diameter of the specimen as:

\[
\text{Diameter } = \frac{\text{? mm}}{\text{? in.}} \quad \text{or} \quad = \frac{\text{? in.}}{\text{? mm}}
\]

So the cross-sectional area is

\[
\text{Area } = \frac{\text{mm}^2}{\text{in.}^2} \quad \text{or} \quad = \frac{\text{in.}^2}{\text{mm}^2}
\]

The bar, made of ______ steel, has a

yield stress of ______ ksi. and an ultimate stress of ______ ksi

For our specimen then, we expect large deformations and failure at a load of

\[
\text{Expected Failure } \sim \frac{\text{lb.}}{\text{KN}} \quad \text{or} \quad = \frac{\text{KN}}{\text{lb.}}
\]

This suggests that our output voltage will range from zero to

\[
\text{Anticipated load voltage at failure } = \frac{\text{volts}}{\text{volts}}
\]

Our goal is to graph how the tensile stress varies with the tensile strain. We need, then to go one step further and construct the factor for tensile stress in terms of voltage. Dividing the force by the area we obtain:

\[
\text{Stress factor } = \frac{\text{Pascals}}{\text{volt}} \quad \text{or} \quad = \frac{\text{psi}}{\text{volt}}
\]

To compute the corresponding values of the strain, we need to measure the change in length of the specimen. The strain is then the ratio of the change in length to the original length.

---

1. 1 Kip is 1000 pounds.
Parameters for determining tensile strain.

An extensometer will be used to measure the change in length between two points located on the surface of the cylindrical specimen. The two knife edges, which are held in place against the specimen by simple elastic bands, are initially spaced a preset distance apart. This “gage length” is

| Gage Length = _______ in | or _______ mm |

The gage factor is determined from “full scale” conditions. The extensometer we will use can accommodate a full scale displacement of +/- 0.2 inches. The full scale output voltage is 4.0 volts. Hence, the scale factor for displacement is

| Displacement factor = _______ in/volt | or _______ mm/volt |

The strain factor is then, dividing by the gage length

| Strain factor = _______ in/in/volt | or _______ mm/mm/volt |

We anticipate, at loading near yielding where the strain is but _______ in/in, the voltage output of the extensometer will be _______ volts.

Data Collection and Analysis

Data will be recorded via the computer (equipped with analogue to digital conversion hardware). An ASCII text file will be produced in three column format. The first dozen or so readings will show negative values for the displacement and load as the specimen “takes up the load”.

| Time (sec) | Displacement (volts) | Load (volts) |

Results

Once we have a plot, we determine the slope in the elastic range. This slope, if taken directly from the graph, will have the units of inch/inch. We follow the trail back through all the scale factors to compute the slope as a ratio of tensile stress to tensile strain. We obtain in this way the following experimentally determined value for $E$, the elastic, or Young’s modulus:

$$E, \text{ the elastic, or Young’s modulus} = \frac{\text{_______}}{\text{_______}} \text{ psi or } \frac{\text{_______}}{\text{_______}} \text{ Pascals}$$

Import the graph from your spreadsheet or simply, physically cut and paste it into this document on the next page.
A graph of the stress-strain curve, ranging up to four times the strain at yield is shown below.

The slope is indicated on the graph.

It differs from the “book value” by ~ %

Reasons for this discrepancy include:

[Here attempt to explain this discrepancy and deviations from linearity.]
2.2 Tension test of a steel rod.

We return to the lab and subject a thin, stainless steel rod to tension. We again measure pairs of values of applied loading and displacement but we do not take the specimen to failure. We will remain in the elastic range, then unload.

Calibration of the LVDT.

We first calibrate the LVDT. [Actually a dc voltage input/output device]. The attached specifications are for the case when the instrument maker's core is used. We use a home-made core which will allow us to load the specimen directly through the LVDT.

We lay the LVDT down on its side, on a sheet of graph paper as shown at the right.

The Hewitt Packard power supply is off.

We make sure the red lead of the LVDT is attached to the plus (+) 20v output terminal, the black lead to the minus (-) 20v terminal. The tracking knob is set all the way around to the right to the "fixed" position.

We attach the clip leads of the digital multi-meter to the blue and green leads of the LVDT. The polarity is not critical. We set the multimeter to the DC volts scale.

After checking with the lab instructor, we turn on the power supply and adjust the plus 20 volt knob to five, 5, volts (read directly from the meter of the power supply) or as close as we can come. Pushing the - 20v meter selection button, we see, because of the tracking feature, a negative voltage of the same magnitude is present at the minus 20v output terminal.

We do not change these settings for the duration of the experiment.

We choose some reference point for the location of the extreme right end of the core such that the LVDT voltage output, as read by the digital meter, is in the vicinity of zero.

We record pairs of LVDT output voltage/core-position values over a volatage range which extends from one side of zero to the other. As per the instrument's specifications,

<table>
<thead>
<tr>
<th>LVDT supply voltage, +/-</th>
<th>volts</th>
</tr>
</thead>
</table>

we note that the working range (with the original core) is specified as

<table>
<thead>
<tr>
<th>LVDT working range = +/-</th>
<th>in</th>
</tr>
</thead>
</table>

hence our calibration data extends over a relative displacement of inches.
The table below shows measured pairs of LVDT output voltage/core position values.

### LVDT Calibration

<table>
<thead>
<tr>
<th>LVDT output</th>
<th>X Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Units?]</td>
<td>[Units?]</td>
</tr>
<tr>
<td>+/-??</td>
<td>+/-??</td>
</tr>
</tbody>
</table>

A plot of the voltage/position pairs is shown below.
A straight line fit to the working range of the instruments yields a slope of

\[ \text{LVDT sensitivity} = \ \text{volts/inch} \]

**Setup of test specimen.**

The schematic at the right shows the test setup. Follow these steps to assemble the arrangement displayed:

Switch off and unplug the power supply, and remove the digital multimeter clip leads from the blue and green outputs of the LVDT. Move the power supply and the LVDT with its red and black leads still attached to the +/- 20 volt output terminals to the top of the beige supply cabinet at the side of the test bed.

We measure the diameter of the specimen... and the length between the heads of the threaded fasteners at each end.

<table>
<thead>
<tr>
<th>Specimen diameter =</th>
<th>+/- ?? [units]</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Specimen length =</th>
<th>+/- ?? [units]</th>
</tr>
</thead>
</table>

Measure also the length of the core which contributes to your measure of strain. (See Appendix).

At this point, make an estimate of a "safe" maximum load for the specimen based upon:

- the yield stress of the tension test observed in the first part of the experiment.

<table>
<thead>
<tr>
<th>Safe Load =</th>
<th>lbs.</th>
</tr>
</thead>
</table>

- the yield stress of 1020 Cold Rolled steel ~90,000 psi

<table>
<thead>
<tr>
<th>Safe Load =</th>
<th>lbs.</th>
</tr>
</thead>
</table>

- the yield stress of 1020 Hot Rolled steel ~ 40,000 psi

<table>
<thead>
<tr>
<th>Safe Load =</th>
<th>lbs.</th>
</tr>
</thead>
</table>
Thread the custom core through the LVDT so that the threaded end of the core protrudes from that end of the LVDT from which all lead wires emerge. With this same end facing the floor, fit the LVDT thru the cylindrical hole in the aluminum piece which has already been attached to the stand. Locate the fixture about midpoint of the LVDT and tighten the wing nut.

Slide the small harness ring over the threaded end of the core, locate the washer, then the nut and turn down the latter until the threads of the core extend beyond the face of the nut.

Now align the fixture and stand such that the test specimen and LVDT are vertical and in line. Do a sighting from two orthogonal directions. You can adjust the relative position of the fixture with respect to the stand and you can adjust the location of the stand on the test bed.

Now plug in the power supply and attach the clip leads of the multimeter to the output leads of the LVDT. Do not turn the power supply on. Have your lab instructor verify the setup.

Adjust the vertical position of the fixture and LVDT relative to the stand such that the reading on the multimeter (DC volts scale) is in the vicinity of zero. Switch to the 400mv DC volt range and adjust further until the meter reads near zero or, better yet, on that side of zero which will approach, and go through zero as load is applied.

Recheck the verticality.

Attach the chain to the harness ring using a “heavy” S hook.

Attach the pail to the other end of the chain hanging below the test bed again using a heavy S hook. Make sure the chain passes through the appropriate slot in the test bed to minimize friction.

The bottom of the pail should hang but a half inch off the mat on the floor. Although the experiment should not take the specimen to failure, we want to make sure that, if failure did occur, the thick end of the custom core does not impact the LVDT from above. Again, have your lab instructor verify the setup.

We now load the specimen, using the big weights, each of which measures

| Unit weight = ____________________ +/- ?? [units] |


We do not exceed an applied load of 70 pounds. We take data upon unloading as well.

**TABLE 1. Tension Test Data**

<table>
<thead>
<tr>
<th>Load</th>
<th>LVDT output [units]</th>
<th>Relative LVDT out [units]</th>
<th>Displcmnt [units]</th>
<th>Strain $\varepsilon$ [units]</th>
<th>Stress $\sigma$ [units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>+/- ???</td>
<td>+/-</td>
<td>+/-</td>
<td>+/-</td>
<td>+/-</td>
<td>+/-</td>
</tr>
<tr>
<td>Chain +pail</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The two columns on the right included data taken in the lab. The remaining columns were computed subsequently.
Results

The plot below shows the stress strain curve obtained. We also redraw the curve obtained for the steel rod on the same plot.

\[
E, \text{ the elastic, or Young's modulus} = \underline{\text{ } } \pm \underline{\text{ ? }} \text{ psi}
\]

The slope of the stress/strain curve in the linear elastic range gives a value for the elastic modulus for the rod of

Again, we see this is (above, below?) the book value by a (significant, small?) amount. This difference (can, can not ?) be accounted for by uncertainty in our making of measurements.

[Here add more discussion]

Conclusions and Recommendations

[Here add your conclusions and recommendations]
Appendix - Uniaxial Tensile Stress, Strain and Linear Springs.

The tension test.

The tension test is a standard test\textsuperscript{1} for characterizing the behavior of a material under uniaxial load. The test consists of pulling on a circular shaft, nominally a centimeter in diameter, and measuring the applied force and the relative displacement of two points on the surface of the shaft in-line with its axis. As the load \( P \) increases from zero on up until the specimen breaks, the relative distance between the two points increases from \( L_0 \) to some final length just before separation. The graph at the right indicates the trace of data points one might obtain for load \( P \) versus \( \Delta \) where

\[
\Delta = L - L_0
\]

If we double the cross-sectional area, \( A \), we expect to have to double the load to obtain the same change in length of the two points on the surface. That indeed is the case. Thus, we can extend our results obtained from a single test on a specimen of cross sectional area \( A \) and length \( L_0 \) to another specimen of the same length but different area if we plot the ratio of load to area, the tensile stress, in place of \( P \).

Similarly, if, instead of plotting the change in length, \( \Delta \), of the two points, we plot the stress against the ratio of the change in length to the original length between the two points our results will be applicable to specimens of varying length. The ratio of change in length to original length is called the extensional strain. We designate the tensile stress by \( \sigma \) and the extensional strain by \( \varepsilon \). The tensile stress is defined as

\[
\sigma = \frac{P}{A}
\]

and the extensional strain by

\[
\varepsilon = \frac{\Delta}{L_0}
\]

The figure below left shows the results of a test of 1020, Cold Rolled Steel. Stress, \( \sigma \) is plotted versus strain \( \varepsilon \). The figure below right shows an abstract representation of the stress-strain behavior as elastic, perfectly plastic material.

\[
\sigma = \frac{P}{A}
\]

Observe:

1. Standard tests for material properties, for failure stress levels, and the like are well documented in the American Society for Testing Materials, ASTM, publications. Go there for the description of how to conduct a tensile test.
• The plot shows a region where the stress is proportional to the strain. The linear relation which holds within this region is written

\[ \sigma = E \cdot \varepsilon \]

where \( E \) is the elastic, or Young's modulus.

• The behavior of the bar in this region is called elastic. Elastic means that when the load is removed, the bar returns to its original, undeformed configuration. That is \( L \) returns to \( L_0 \). There is no permanent set.

• The relative displacements of points — the strains in the elastic region — are very small, generally insensible without instruments to amplify their magnitude. To “see” a relative displacement of two points originally 100 mm apart when the stress is on the order of 400 Mega Newtons/m² your eyes would have to be capable of resolving a relative displacement of the two points of 0.2 mm! Strains in most structural materials are on the order of tenths of a percent at most.

• At some stress level, the bar does not return to its undeformed shape after removing the load. This stress level is called the yield strength. The yield strength defines the limit of elastic behavior; beyond the yield point the material behaves plastically. In the graphs above, we show the yield strength defined at a 2% offset, that is, as the intersection of the experimentally obtained stress-strain curve with a straight line of slope, \( E \), intersecting the strain axis at a strain of 0.002. Its value is approximately 600 MN/m².

• Loading of the bar beyond the yield strength engenders very large relative displacements for relatively small further increments in the stress, \( \sigma \).

A solid bar as a linear spring.

For a linear spring, we write \( F = k \cdot \Delta \) where \( k \) is the spring stiffness and has the dimensions force per length.

We can treat our solid bar, subject to uniaxial tension, as a linear spring if we substitute for the strain, its representation in terms of \( \Delta \) and \( L \) (drop the subscript 0), and for the stress, its representation in terms of \( F \) and \( A \), the cross-sectional area. We obtain:

\[ F = (AE/L) \cdot \Delta = k \cdot \Delta \]
Two linear springs in series ~ a tensile test specimen + elastic test machine.

If we join two linear springs of different stiffness in series, we can, using the principles of solid mechanics, determine the equivalent stiffness of the combination.

From equilibrium considerations, the force in each of the springs is the same, \( F \). Their extensions, \( \Delta_1 \) and \( \Delta_2 \), are different; from the constitutive equations, we have

\[
\Delta_2 = \frac{F}{k_2} \quad \text{and} \quad \Delta_1 = \frac{F}{k_1}
\]

And compatibility of deformation requires

\[
\Delta = \Delta_1 + \Delta_2
\]

From all of this we derive an expression for the equivalent system of the whole system, namely

\[
F = K \cdot \Delta \quad \text{where} \quad K = \frac{k_1 \cdot k_2}{k_1 + k_2} = \frac{k_2}{1 + k_2/k_1}
\]