MACRO DESIGN MODELS FOR A SINGLE ROUTE

Outline
1. Introduction to analysis approach
2. Bus frequency model
3. Stop/station spacing model
Introduction to Analysis Approach

- Basic approach is to establish an aggregate total cost function including:
  - operator cost as $f(\text{design parameters})$
  - user cost as $g(\text{design parameters})$

- Minimize total cost function to determine optimal design parameter (s.t. constraints)

Variants include:
- Maximize service quality s.t. budget constraint
- Maximize consumer surplus s.t. budget constraint
Bus Frequency Model: the Square Root Model

Problem: define bus service frequency on a route as a function of ridership

Total Cost = operator cost + user cost

\[ Z = c \cdot \frac{t}{h} + b \cdot r \cdot \frac{h}{2} \]

where
- \( Z \) = total (operator + user) cost per unit time
- \( c \) = operating cost per unit time
- \( t \) = round trip time
- \( h \) = headway – the decision variable to be determined
- \( b \) = value of unit passenger waiting time
- \( r \) = ridership per unit time

Minimizing \( Z \) w.r.t. \( h \) yields:

\[ h = \sqrt{\frac{2ct}{br}} \quad \text{or} \quad \sqrt{2 \left( \frac{c}{b} \right) \left( \frac{t}{r} \right)} \]
This is the Square Rule with the following implications:

- high frequency is appropriate where (cost of wait time/cost of operations time) is high
- frequency is proportional to the square root of ridership per unit time for routes of similar length

Frequency-Ridership Relationship
Square Root Model (cont’d)

- load factor is proportional to the square root of the product of ridership and route length.

![Bus Capacity-Ridership Relationship](image)

- Load Factor $>1$
- $t_1 > t_2$
- Passengers/bus
- Bus Capacity
- Ridership
Critical Assumptions:
• bus capacity is never binding
• wait time savings are only frequency benefits
• ridership $\neq f$ (frequency)
• simple wait time model
• budget constraint is not binding

Possible Remedies:
• introduce bus capacity constraint
• modify objective function
• introduce $r=f(h)$ and re-define objective function
• modify objective function
• introduce budget constraint
**Bus Frequency Example**

If:  
\[ c = \$90/\text{bus hour}, \]
\[ b = \$10/\text{passenger hour}. \]
\[ t = 90 \text{ mins}, \]
\[ r = 1000 \text{ passengers/hour}, \]

Then:  
\[ h_{OPT} = 11 \text{ mins} \]
Problem: determine optimal stop or station spacing

Trade-off is between walk access time (which increases with station spacing), and in-vehicle time (which decreases as station spacing increases) for the user, and operating cost (which decreases as station spacing increases)

Define $Z = \text{total cost per unit distance along route and per headway}$

and $T_{st} = \text{time lost by vehicle making a stop}$

$c = \text{vehicle operating cost per unit time}$

$s = \text{station/stop spacing - the decision variable to be determined}$

$N = \text{number of passengers on board vehicle}$

$v = \text{value of passenger in-vehicle time}$

$D = \text{demand density in passenger per unit route length per headway}$

$v_{acc} = \text{value of passenger access time}$

$w = \text{walk speed}$

$c_s = \text{station/stop cost per headway}$
Stop/Station Spacing Model (cont’d)

\[ Z = \frac{T_{st}}{s} (c + N \cdot v) + \frac{c_s}{s} + \frac{s}{4} \cdot D \cdot \frac{v_{acc}}{w} \]

Minimizing \( Z \) w.r.t. \( s \) gives:

\[ s_{OPT} = \left[ \frac{4w}{Dv_{acc}} \left( c_s + T_{st} (c_v + Nv) \right) \right]^{1/2} \]

Yet another square root relationship, implying that station/stop spacing increases with:

- walk speed
- station/stop cost
- time lost per stop
- vehicle operating cost
- number of passengers on board vehicle
- value of in-vehicle time

and decreases with:

- demand density
- value of access time
Bus Stop Spacing

**U.S. Practice**
- 200 m between stops (8 per mile)
- shelters are rare
- little or no schedule information

**European Practice**
- 320 m between stops (5 per mile)
- named & sheltered
- up to date schedule information
- scheduled time for every stop
Stop Spacing Tradeoffs

- Walking time
- Riding time
- Operating cost
- Ride quality

![Graph showing tradeoffs between stop spacing, walking time, riding time, operating cost, and total extra cost.](image-url)
Walk Access: Block-Level Modeling

Figure by MIT OCW.
Results: MBTA Route 39*

AM Peak Inbound results

- Avg walking time up 40 s
- Avg riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses