OUTLINE

• Hierarchy of choices
• Level of service attributes
• Estimating the Transfer Penalty*
• Modeling issues
  -- data availability
  -- logit revisited

HIERARCHY OF CHOICES

A. Long-Term Decisions: made infrequently by any household
   • where to live
   • where to work
     *Transport is one component of these choices*

B. Medium-Term Decisions
   • household vehicle ownership
   • mode for journey to work
     *Transport is central to these choices*

C. Short-Term Decisions
   • daily activity and travel choices:
     What, where, when, for how long, and in what order, by which mode and route
     *Transport is important for these choices*
LEVEL OF SERVICE ATTRIBUTES

A. Important but hard to quantify:
   • flexibility
   • privacy
   • status
   • enjoyment/happiness/well-being
   • comfort
   • safety and security
   • reliability
LEVEL OF SERVICE ATTRIBUTES

B. Important but easier to quantify:
   • travel time
     • wait time
     • in-vehicle time
     • walk time
   • transfers
   • cost
     • out of pocket
LEVEL OF SERVICE ATTRIBUTES

Difficulties:

• differences in values among individuals
• objective measures may differ from perceptions
• tendency to focus on what can be measured
• hard to appraise reactions to a very different alternative
ASSESSING THE TRANSFER PENALTY: A GIS-BASED DISAGGREGATE MODELING APPROACH

Outline

- Objectives
- Prior Research
- Modeling Approach
- Data Issues
- Model Specifications
- Analysis and Interpretation
- Conclusions

OBJECTIVES

• Improve our understanding of how transfers affect behavior

• Estimate the impact of each variable characterizing a transfer

• Identify transfer attributes which can be improved cost-effectively
## Previous Transfer Penalty Results

<table>
<thead>
<tr>
<th>Previous Studies</th>
<th>Variables in the Utility Function</th>
<th>Transfer Types (Model Structure)</th>
<th>Transfer Penalty Equivalence</th>
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</thead>
<tbody>
<tr>
<td>Alger et al, 1971 Stockholm</td>
<td>Walking time to stop Initial waiting time Transit in-vehicle time Transit cost</td>
<td>Subway-to-Subway Rail-to-Rail Bus-to-Rail Bus-to-Bus</td>
<td>4.4 minutes in-vehicle time 14.8 minutes in-vehicle time 23.0 minutes in-vehicle time 49.5 minutes in-vehicle time</td>
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<td>Han, 1987 Taipei, Taiwan</td>
<td>Initial waiting time Walking time to stop In-vehicle time Bus fare Transfer constant</td>
<td>Bus-to-Bus (Path Choice)</td>
<td>30 minutes in-vehicle time 10 minutes initial wait time 5 minutes walk time</td>
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<td>Hunt, 1990 Edmonton, Canada</td>
<td>Transfer Constant Walking distance Total in-vehicle time Waiting time Number of transfers</td>
<td>Bus-to-Light Rail (Path Choice)</td>
<td>17.9 minutes in-vehicle time</td>
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## Previous Transfer Penalty Results (cont'd)

<table>
<thead>
<tr>
<th>Previous Studies</th>
<th>Variables in the Utility Function</th>
<th>Transfer Types (Model Structure)</th>
<th>Transfer Penalty Equivalence</th>
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</thead>
<tbody>
<tr>
<td>Liu, 1997 New Jersey, NJ</td>
<td>Transfer Constant In-vehicle time Out-of-vehicle time One way cost Number of transfers</td>
<td>Auto-to-Rail Rail-to-Rail (Modal Choice)</td>
<td>15 minutes in-vehicle time 1.4 minutes in-vehicle time</td>
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<tr>
<td>CTPS, 1997 Boston, MA</td>
<td>Transfer Constant In-vehicle time Walking time Initial waiting time Transfer waiting time Out-of-vehicle time Transit fare</td>
<td>All modes combined (Path and Mode Choice)</td>
<td>12 to 15 minutes in-vehicle time</td>
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<tr>
<td>Wardman, Hine and Stradling, 2001</td>
<td>Utility function not specified</td>
<td>Bus-to-Bus Auto-to-Bus Rail-to-Rail</td>
<td>4.5 minutes in-vehicle time 8.3 minutes in-vehicle time 8 minutes in-vehicle time</td>
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</tbody>
</table>
PRIOR RESEARCH – A CRITIQUE

• Wide range of transfer penalty
• Incomplete information on path attributes
• Limited and variable information on transfer facility attributes
• Some potentially important attributes omitted
MODELING APPROACH

• Use standard on-board survey data including:
  -- actual transit path including boarding and alighting locations
  -- street addresses of origin and destination
  -- demographic and trip characteristics

• Focus on respondents who:
  -- travel to downtown Boston destinations by subway
  -- have a credible transfer path to final destination
MODELING APPROACH

- Define transfer and non-transfer paths to destination from subway line accessing downtown area

- For each path define attributes:
  -- walk time    -- transfer walk time
  -- in-vehicle time    -- transfer wait time

- Specify and estimate binary logit models for probability of selecting transfer path
TWO OPTIONS TO REACH THE DESTINATION

Subway line used to access downtown Boston

Non-transfer path

Transfer path

1) Subway to Station A
2) Walk to D

Transfer Station

Destination

Non-transfer Path
1) Subway to Station A
2) Walk to D

Transfer Path
1) Subway to Station B
2) Transfer at B
3) Subway to Station C
4) Walk to D

Figure by MIT OCW.
MBTA SUBWAY CHARACTERISTICS

- Three heavy rail transit lines (Red, Orange, and Blue)
- One light rail transit line (Green)
- Four major downtown subway transfer stations (Park, Downtown Crossing, Government Center, and State)
- 21 stations in downtown study area
- Daily subway ridership: 650,000
- Daily subway-subway transfers: 126,000
THE MBTA SUBWAY IN DOWNTOWN BOSTON

Downtown Boston and the MBTA Subway System

- Transfer Stations
- MBTA Stations
- MBTA Subway Lines
  - Blue
  - Green
  - Orange
  - Red
  - Roads
  - Boston Common
  - Beacon Hill
  - Water
  - Boston

Figure by MIT OCW.
DATA ISSUES

• Data from 1994 MBTA on-board subway survey
• 38,888 trips in the dataset
• 15,000 geocodable destination points
• 6,500 in downtown area
• 3,741 trips with credible transfer option based on:
  • closest station is not on the subway line used to enter the downtown area
• 67% of trips with credible transfer option actually selected non-transfer path
• 3,140 trips used for model estimation
VARIABLES

A Transit Path Variables

• Walk time savings: based on shortest path and assume 4.5 km per hour walk speed
• Extra in-vehicle time: based on scheduled trip time

B Transfer Attributes

• Transfer walk time
• Transfer wait time: half the scheduled headway
• Assisted change in level: a binary variable with value 1 if there is an escalator
C. Pedestrian Environment Variables

- Land use: difference in Pedestrian Friendly Parcel (PFP) densities
- Pedestrian Infrastructure Amenity: difference in average sidewalk width
- Open Space: a trinary variable reflecting walking across Boston Common
- Topology: a trinary variable reflecting walking through Beacon Hill

D. Trip and Demographic Variables
THE SEQUENCE OF MODEL DEVELOPMENT

<table>
<thead>
<tr>
<th>Simple Model</th>
<th>Transfer Effects in the System</th>
<th>Pedestrian Environmental Effects outside the System</th>
<th>Trip and Personal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Constant, Walk Time Saving</td>
<td>+ Extra In-vehicle Time</td>
<td>+ Transfer Walk Time, Transfer Wait Time, Assisted Level Change,</td>
<td></td>
</tr>
<tr>
<td>B Constant, Total Time Saving</td>
<td>+ Station Dummies</td>
<td>B + C</td>
<td>F + Income, Gender, Occupation, Purpose, etc</td>
</tr>
<tr>
<td></td>
<td>Transfer Station Effects</td>
<td></td>
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</tbody>
</table>

Figure by MIT OCW.
MODEL A: SIMPLEST MODEL

Specification

• Assume every transfer is perceived to be the same
• Only two variables
  -- transfer constant
  -- walk time savings

Findings

• A transfer is perceived as equivalent to 9.5 minutes of walking time
## MODEL A RESULTS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>t statistics</th>
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</thead>
<tbody>
<tr>
<td>Transfer Constant</td>
<td>-2.39</td>
<td>-28.57</td>
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<tr>
<td>Walk Time Savings (minutes)</td>
<td>0.25</td>
<td>20.78</td>
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<td># of Observations</td>
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<tr>
<td>Final log-likelihood</td>
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</tr>
<tr>
<td>Adjusted $\rho^2$</td>
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<td>0.309</td>
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</table>
MODEL B: TRANSFER STATION SPECIFIC MODEL

Specification
• Assume each transfer station is perceived differently
• Variables are:
  -- walk time savings
  -- extra in-vehicle time
  -- station-specific transfer dummies

Findings
• Improved explanatory power (over Model A)
• Transfer stations are perceived differently
• Park is the best (4.8 minutes of walk time equivalence)
• State is the worst (9.7 minutes of walk time equivalence)
MODEL B RESULTS

<table>
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<th>Variables</th>
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</table>
MODEL C: TRANSFER ATTRIBUTES MODEL

Specification

• Transfer attributes affect transfer perceptions:
  -- transfer walk time
  -- transfer wait time
  -- assisted change in level

Findings

• Improved explanatory power (over Model B)
• Residual transfer penalty is equivalent to 3.5 minutes of walking time savings
• Transfer waiting time is least significant
## MODEL C RESULTS

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<td>0.385</td>
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</table>
MODEL D: COMBINED ATTRIBUTE & STATION MODEL

Specification
• Combines the variables in Model B and C
• Estimates separate models for peak and off-peak periods

Findings
• Improved explanatory power (over Model C)
• Government Center is perceived as worse than other transfer stations
• Residual transfer penalty in off-peak period at other transfer stations vanishes
• In the peak period model the transfer waiting time is not significant
## MODEL D RESULTS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
<th>Model D</th>
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<td>0.369</td>
<td>0.385</td>
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</tbody>
</table>

Note, ***: $P < 0.001$; **: $P < 0.05$; *: $P < 0.1$
MODEL E: PEDESTRIAN ENVIRONMENT MODEL

Specification

- Better pedestrian environment should lead to greater willingness to walk
- Add pedestrian environment variables to Model D

Findings

- Improved explanatory power (over Model D)
- Greater sensitivity to pedestrian environment in off-peak model
- Both Boston Common (positively) and Beacon Hill (negatively) affect transfer choices as expected
- Pedestrian environment variables can affect the transfer penalty by up to 6.2 minutes of walking time equivalence
## MODEL E RESULTS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
<th>Model D</th>
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<td>Note, ***: $P &lt; 0.001$; **: $P &lt; 0.05$; *: $P &lt; 0.1$</td>
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</table>
ANALYSIS AND INTERPRETATION

• The transfer penalty has a range rather than a single value
• The attributes of the transfer explain most of the variation in the transfer penalty
• For the MBTA subway system the transfer penalty varies between the equivalent of 2.3 minutes and 21.4 minutes of walking time
• Model results are consistent with prior research findings
### RANGE OF THE TRANSFER PENALTY

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Underlying Variables</th>
<th>Adjusted $\rho^2$</th>
<th>The Range of the Penalty (Equivalent Value of )</th>
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<tbody>
<tr>
<td>A</td>
<td>Transfer constant</td>
<td>0.309</td>
<td>9.5 minutes of walking time</td>
</tr>
<tr>
<td>B</td>
<td>Government Center Downtown Crossing State</td>
<td>0.369</td>
<td>4.8 ~ 9.7 minutes of walking time</td>
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<tr>
<td>C</td>
<td>Transfer constant • Transfer walk time • Transfer wait time • Assisted Level Change</td>
<td>0.385</td>
<td>4.3 ~ 15.2 minutes of walking time</td>
</tr>
<tr>
<td>D</td>
<td>Transfer constant • Transfer walk time • Transfer wait time • Assisted Level Change • Government Center</td>
<td>0.414 (Peak) 0.357 (Off-peak)</td>
<td>4.4 ~ 19.4 minutes of walking time (Peak) 2.3 ~ 21.4 minutes of walking time (Off-peak)</td>
</tr>
<tr>
<td>Studies</td>
<td>Alger et al 1971</td>
<td>Liu 1997</td>
<td>Wardman et al 2001</td>
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<td>------------------</td>
<td>----------</td>
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<td>City</td>
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<td>Value of the</td>
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<tr>
<td>Transfer Penalty*</td>
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</tr>
</tbody>
</table>

* Minutes of in-vehicle time
LIMITATIONS OF RESEARCH

• Findings relate only to current transit riders
• Only subway-subway transfer studied
  -- no transfer payment involved
  -- transfers are protected from weather
  -- headways are very low
• Weather variable not included
SOURCES OF DATA ON USER BEHAVIOR

• Revealed Preference Data
  – Travel Diaries
  – Field Tests

• Stated Preference Data
  – Surveys
  – Simulators
STATED PREFERENCES / CONJOINT EXPERIMENTS

- Used for product design and pricing
  -- For products with significantly different attributes
  -- When attributes are strongly correlated in real markets
  -- Where market tests are expensive or infeasible

Uses data from survey “trade-off” experiments in which attributes of the product are systematically varied

Applied in transportation studies since the early 1980s
Objective is to make aggregate predictions from

-- A disaggregate model, \( P(i | X_n) \)

-- Which is based on individual attributes and characteristics, \( X_n \)

-- Having only limited information about the explanatory variables
THE AGGREGATE FORECASTING PROBLEM

• The fraction of population $T$ choosing alt. $i$ is:

$$W(i) = \int P(i \mid X) p(X) dX$$

, $p(X)$ is the density function of $X$

$$= \frac{1}{N_T} \sum_{n=1}^{N_T} P(i \mid X_n)$$

, $N_T$ is the # in the population of interest

• Not feasible to calculate because:
  -- We never know each individual’s complete vector of relevant attributes
  -- $p(X)$ is generally unknown

• The problem is to reduce the required data
SAMPLE ENUMERATION

• Use a sample to represent the entire population

• For a random sample:

\[ \hat{W}(i) = \frac{1}{N_s} \sum_{n=1}^{N_s} \hat{P}(i \mid x_n) \]

where \( N_s \) is the \# of obs. in sample

• For a weighted sample:

\[ \hat{W}(i) = \sum_{n=1}^{N_s} \sum_{n} \frac{W_n}{W_n} \hat{P}(i \mid x_n) \]

where \( \frac{1}{W_n} \) is \( x_n \)'s selection prob.

• No aggregation bias, but there is sampling error
DISAGGREGATE PREDICTION

Generate a representative population

Apply demand model
- Calculate probabilities or simulate decision for each decision maker
- Translate into trips
- Aggregate trips to OD matrices

Assign traffic to a network

Predict system performance
GENERATING DISAGGREGATE POPULATIONS

Household surveys

Exogenous forecasts

Census data

Counts

Data fusion (e.g., IPF, HH evolution)

Representative Population
LOGIT MODEL PROPERTY AND EXTENSION

- Independence from Irrelevant Alternatives (IIA) property -- Motivation for Nested Logit
- Nested Logit - specification and an example
INDEPENDENCE FROM IRRELEVANT ALTERNATIVES (IIA)

• Property of the Multinomial Logit Model
  – \( \varepsilon_{jn} \) independent identically distributed (i.i.d.)
  – \( \varepsilon_{jn} \sim \text{ExtremeValue}(0, \mu) \) \( \forall j \)

\[
P_n \left( i \mid C_n \right) = \frac{e^{\mu V_{in}}}{\sum_{j \in C_n} e^{\mu V_{jn}}}
\]

so

\[
\frac{P \left( i \mid C_1 \right)}{P \left( j \mid C_1 \right)} = \frac{P \left( i \mid C_2 \right)}{P \left( j \mid C_2 \right)} \quad \forall i, j, C_1, C_2
\]

such that \( i, j \in C_1, i, j \in C_2, C_1 \subseteq C_n \) and \( C_2 \subseteq C_n \)
EXAMPLES OF IIA

- Route choice with an overlapping segment

\[
P(1 \mid \{1, 2a, 2b\}) = P(2a \mid \{1, 2a, 2b\}) = P(2b \mid \{1, 2a, 2b\}) = \frac{e^{\mu T}}{\sum_{j \in \{1, 2a, 2b\}} e^{\mu T}} = \frac{1}{3}
\]
RED BUS / BLUE BUS PARADOX

- Consider that initially auto and bus have the same utility
  - \( C_n = \{\text{auto, bus}\} \) and \( V_{\text{auto}} = V_{\text{bus}} = V \)
  - \( P(\text{auto}) = P(\text{bus}) = 1/2 \)

- Now suppose that a new bus service is introduced that is identical to the existing bus service, except the buses are painted differently (red vs. blue)
  - \( C_n = \{\text{auto, red bus, blue bus}\}; \ V_{\text{red bus}} = V_{\text{blue bus}} = V \)
  - MNL now predicts
    - \( P(\text{auto}) = P(\text{red bus}) = P(\text{blue bus}) = 1/3 \)
  - We’d expect
    - \( P(\text{auto}) = 1/2, \ P(\text{red bus}) = P(\text{blue bus}) = 1/4 \)
IIA AND AGGREGATION

- Divide the population into two equally-sized groups: those who prefer autos, and those who prefer transit.

- Mode shares before introducing blue bus:

<table>
<thead>
<tr>
<th>Population</th>
<th>Auto Share</th>
<th>Red Bus Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto people</td>
<td>90%</td>
<td>10%</td>
</tr>
<tr>
<td>Transit people</td>
<td>10%</td>
<td>90%</td>
</tr>
<tr>
<td>Total</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

\[
P(\text{auto})/P(\text{red bus}) = 9 \quad \text{and} \quad P(\text{auto})/P(\text{red bus}) = 1/9
\]

- Auto and red bus share ratios remain constant for each group after introducing blue bus:

<table>
<thead>
<tr>
<th>Population</th>
<th>Auto Share</th>
<th>Red Bus Share</th>
<th>Blue Bus Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto people</td>
<td>81.8%</td>
<td>9.1%</td>
<td>9.1%</td>
</tr>
<tr>
<td>Transit people</td>
<td>5.2%</td>
<td>47.4%</td>
<td>47.4%</td>
</tr>
<tr>
<td>Total</td>
<td>43.5%</td>
<td>28.25%</td>
<td>28.25%</td>
</tr>
</tbody>
</table>
• Overcome the IIA Problem of Multinomial Logit when
  -- Alternatives are correlated
    (e.g., red bus and blue bus)
  -- Multidimensional choices are considered
    (e.g., departure time and route)
• Example: Mode Choice (Correlated Alternatives)
TREE REPRESENTATION OF NESTED LOGIT

- Example: Route and Departure Time Choice (Multidimensional Choice)
NESTED MODEL ESTIMATION

- Logit at each node
- Utilities at lower level enter at the node as the *inclusive* value

\[ I_{NM} = \ln \left( \sum_{i \in C_{NM}} e^{V_i} \right) \]

- The inclusive value is often referred to as *logsum*
NESTED MODEL - EXAMPLE

\[
P(i \mid NM) = \frac{e^{V_i}}{e^{V_{Walk}} + e^{V_{Bike}}}, \quad i = \text{Walk, Bike}
\]

\[
I_{NM} = \ln(e^{V_{Walk}} + e^{V_{Bike}})
\]
NESTED MODEL - EXAMPLE

\[ P(i \mid M) = \frac{e^{V_i}}{e^{V_{\text{Car}}} + e^{V_{\text{Taxi}}} + e^{V_{\text{Bus}}}} \quad i = \text{Car, Taxi, Bus} \]

\[ I_M = \ln(e^{V_{\text{Car}}} + e^{V_{\text{Taxi}}} + e^{V_{\text{Bus}}}) \]
NESTED MODEL - EXAMPLE

\[
P(NM) = \frac{e^{a_{NM} + \gamma I_{NM}}}{e^{a_{NM} + \gamma I_{NM}} + e^{\gamma I_{M}}}
\]

\[
P(M) = \frac{e^{\gamma I_{M}}}{e^{a_{NM} + \gamma I_{NM}} + e^{\gamma I_{M}}}
\]
NESTED MODEL - EXAMPLE

- Calculation of choice probabilities

\[ P(Bus) = P(Bus \mid M) \cdot P(M) \]

\[ = \frac{e^{V_{Bus}}}{e^{V_{Car}} + e^{V_{Taxi}} + e^{V_{Bus}}} \cdot \frac{e^{\gamma I_M}}{e^{a_{NM} + \gamma I_{NM}} + e^{\gamma I_M}} \]

\[ = \frac{e^{V_{Bus}}}{e^{V_{Car}} + e^{V_{Taxi}} + e^{V_{Bus}}} \cdot \frac{e^{\gamma \ln(e^{V_{Car}} + e^{V_{Taxi}} + e^{V_{Bus}})}}{e^{a_{NM} + \gamma \ln(e^{V_{Walk}} + e^{V_{Bike}}) + e^{\gamma \ln(e^{V_{Car}} + e^{V_{Taxi}} + e^{V_{Bus}})}}} \]