Introduction to Transportation Demand Analysis and Overview of Consumer Theory

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Transportation Systems Analysis: Demand & Economics
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Part One: Introduction to Transportation Demand Analysis

Outline

I. Introduction to Transportation Demand Analysis
   • Choices
   • Complexity
   • Sample statistics
   • Roles of demand models

II. Overview of Consumer Theory
Choices Impacting Transport Demand

● Decisions made by Organizations
  – Firm locates in Boston or Waltham
  – Firm invests in home offices, high speed connections
  – Developer builds in downtown or suburbs

● Decisions made by Individual/Households
  – Live in mixed use area in Boston or in residential suburb
  – Do not work or work (and where to work)
  – Own a car or a bike
  – Own an in-vehicle navigation system
  – Work Monday-Friday 9-5 or work evenings and weekends
  – Daily activity and travel choices: what, where, when, for how long, in what order, by which mode and route, using what telecommunications
Complexity of Transport Demand

- Valued as input to other activities (derived demand)
- Encompasses many interrelated decisions
  - Very long-term to very short-term
- Large number of distinct services differentiated by location and time
- Demographics & socioeconomic matter
- Sensitivity to service quality
- Supply and demand interact via congestion

*Complexity* and *Variety* → wide assortment of models to analyze transportation users’ behavior.
## Mode Share Statistics

### Transit Shares for Work Trips in Selected U.S. Cities

<table>
<thead>
<tr>
<th>City</th>
<th>Year</th>
<th>Transit Mode Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston, MA</td>
<td>1990</td>
<td>10.64</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>9.03</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td>1990</td>
<td>13.66</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>11.49</td>
</tr>
<tr>
<td>New York, NY</td>
<td>1990</td>
<td>26.57</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>24.90</td>
</tr>
<tr>
<td>Houston, TX</td>
<td>1990</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>3.28</td>
</tr>
<tr>
<td>Phoenix, AZ</td>
<td>1990</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Travel Expenditures

- The average generalized cost (money and time) per person in developed countries is very stable.

**Transport Demand Elasticities**

- **Elasticity:** % change in demand resulting from 1% change in an attribute

- Derived from demand models:

<table>
<thead>
<tr>
<th>Work Trips (San Francisco)</th>
<th>Auto</th>
<th>Bus</th>
<th>Rail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>-0.47</td>
<td>-0.58</td>
<td>-0.86</td>
</tr>
<tr>
<td>In-vehicle time</td>
<td>-0.22</td>
<td>-0.60</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vacation Trips (U.S.)</th>
<th>Auto</th>
<th>Bus</th>
<th>Rail</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>-0.45</td>
<td>-0.69</td>
<td>-1.20</td>
<td>-0.38</td>
</tr>
<tr>
<td>Travel time</td>
<td>-0.39</td>
<td>-2.11</td>
<td>-1.58</td>
<td>-0.43</td>
</tr>
</tbody>
</table>

Value of Time

- The monetary value of a unit of time for a user.

### Work Trips (San Francisco)

<table>
<thead>
<tr>
<th></th>
<th>Auto</th>
<th>Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-vehicle time</td>
<td>140</td>
<td>76</td>
</tr>
<tr>
<td>Walk access time</td>
<td>273</td>
<td></td>
</tr>
<tr>
<td>Transfer wait time</td>
<td>195</td>
<td></td>
</tr>
</tbody>
</table>

### Vacation Trips (U.S.)

<table>
<thead>
<tr>
<th></th>
<th>Auto</th>
<th>Bus</th>
<th>Rail</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total travel time</td>
<td>6</td>
<td>79-87</td>
<td>54-69</td>
<td>149</td>
</tr>
</tbody>
</table>

### Freight

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total transit time</td>
<td>6-21</td>
<td>8-18</td>
</tr>
</tbody>
</table>

Role of Demand Models

- Forecasts, parameter estimates, elasticities, values of time, and consumer surplus measures obtained from demand models are used to improve understanding of the ramifications of alternative investment and policy decisions.

- Many uncertainties affect transport demand and the models are about to do the impossible.
Role of Demand Models: Examples

● From Previous Lecture:
  – High Speed Rail: Works in Japan/Europe, how about in US?
  – Traffic Jams: Build more, manage better, or encourage transit use?
  – Truck Traffic: evaluate tradeoffs of environmental protection.
Part Two: Overview of Consumer Theory

Outline

- Basic concepts
  - Preferences
  - Utility
  - Choice
- Additional details important to transportation
- Relaxation of assumptions
- Appendix: Dual concepts in demand analysis
Preferences

- The consumer is faced with a set of possible consumption bundles
  - Consumption bundle: a vector of quantities of different products and services \( X = \{x_1, \ldots, x_i, \ldots, x_m\} \)
  - A bundle is an array of consumption amounts of different goods
- Preferences: ordering of the bundles
  - \( X \succeq Y \): Bundle \( X \) is preferred to \( Y \)
  - Behavior: choose the most preferred consumption bundle
  - Transitivity, Completeness, and Continuity
The Utility Function

- A function that represents the consumer’s preferences ordering $X \succ Y \iff U(X) > U(Y)$
  - Utility function is not unique
    - $U(x_1, x_2) = ax_1 + bx_2$
    - $U(x_1, x_2) = x_1^a x_2^b$
  - Unaffected by order-preserving transformation
    - $10 \times U(x_1, x_2) + 10$?
    - $\exp(U(x_1, x_2))$?
    - $(U(x_1, x_2))^2$?
Indifference Curves

- Constant utility curve
- The consumer is indifferent among different bundles on the same curve
Marginal Utility and Trade-offs

- Marginal utility
  \[ MU_i = \frac{\partial U(X)}{\partial x_i} \]
  \[
  U(x_1, x_2) = 4x_1 + 2x_2 \\
  MU_1 = 4
  \]
  \[
  U(x_1, x_2) = x_1^{0.5} x_2^{0.5} \\
  MU_1 = 0.5x_1^{-0.5} x_2^{0.5}
  \]

- Marginal rate of substitution (MRS)
  \[
  MRS = \frac{\frac{\partial U(X)}{\partial x_1}}{\frac{\partial U(X)}{\partial x_2}} = \frac{MU_1}{MU_2}
  \]
  \[
  MRS = \frac{4}{2} \\
  MRS = \frac{x_2}{x_1}
  \]
Consumer Behavior

- Utility (preference) maximization
- Bounded by the available income

\[
\max U(X)
\]

\[
s.t. \quad PX \leq I \quad (\sum_{i=1}^{m} p_i x_i \leq I)
\]

\[
X \quad \text{feasible} \quad (\text{e.g. non-negativity})
\]

- \( P \) - vector of prices \( P = \{p_1, \ldots, p_i, \ldots, p_m\} \)
- \( I \) - income

- When considering two goods, the constraint would be:

\[
p_1 x_1 + p_2 x_2 \leq I
\]
Geometry of the Consumer’s Problem

\[ \frac{I}{p_2} \]

Income constraint

\[ \frac{I}{p_1} \]

Slope

\[ = \frac{-p_1}{p_2} \]

Indifference curves

\[ x_1^* \]

\[ x_2^* \]
Revealed Preferences

- A chosen bundle is revealed preferred to all other feasible bundles:

\[ X(P^0, I^0) \] - the demanded bundle at Point 0

\[ X(P^0, I^0) \succ X(P^1, I^1) \] if \[ P^0 X(P^0, I^0) \geq P^0 X(P^1, I^1) \]
Indirect Revealed Preferences

- Transitivity of preferences:
  - \( X(P^0, I^0) \) is indirectly revealed preferred to \( X(P^2, I^2) \) if:
    \[
    X(P^0, I^0) \succ X(P^1, I^1) \quad \text{and} \quad X(P^1, I^1) \succ X(P^2, I^2)
    \]
Optimal Consumption

- The consumer’s problem
  \[
  \max U(X) \\
  s.t. \quad PX \leq I
  \]

- Assuming \( U(X) \) increases with \( X \)
  \[
  PX = I
  \]

- The Lagrangean
  \[
  L(X, \lambda) = U(X) + \lambda(I - PX)
  \]
  \( \lambda \): Lagrange multiplier of budget constraint
Optimal Consumption

- Optimality conditions:

\[
\frac{\partial L}{\partial x_i} = \frac{\partial U(X^*)}{\partial x_i} - \lambda p_i = 0 \quad \forall i = 1,\ldots,m
\]

\[
\Rightarrow \quad \frac{\partial U(X^*)}{\partial x_i} = \lambda p_i \quad \forall i
\]

- Dividing conditions:

\[
\underbrace{\frac{\partial U(X^*)}{\partial x_i}}_{MRS} = \frac{p_i}{p_j} \quad \forall i \neq j
\]
Example: Cobb-Douglas Utility

● The consumer’s problem:

\[ \max U(X) = x_1^a x_2^b \]
\[ \text{s.t. } p_1 x_1 + p_2 x_2 \leq I \]

● The Lagrangean: \[ L(X, \lambda) = x_1^a x_2^b + \lambda(I - p_1 x_1 - p_2 x_2) \]

● Optimal solution:

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= a x_1^{a-1} x_2^b - \lambda p_1 = 0 \\
\frac{\partial L}{\partial x_2} &= b x_1^a x_2^{b-1} - \lambda p_2 = 0 \\
\frac{\partial L}{\partial \lambda} &= I - p_1 x_1^* - p_2 x_2^* = 0
\end{align*}
\]

\[
\begin{align*}
x_1^* &= \frac{a}{a+b} \frac{I}{p_1} \\
x_2^* &= \frac{b}{a+b} \frac{I}{p_2}
\end{align*}
\]
Review

Basic concepts
- Preferences
- Utility functions \( U(X) \)
- Optimal consumption \( \max U(X) \text{ s.t. } PX \leq I \)
- Demand functions \( X^* = X(P,I) \)

Next…
- Indirect utility
- Complements and Substitutes
- Elasticity
- Consumer surplus
Indirect Utility Function

- Recall
  - \( X^* \) - the demanded bundle
  - \( X(P, I) \) - the consumer’s demand function
- Indirect utility function
  - The maximum utility achievable at given prices and budget:
    \[
    V(P, I) = \max U(X)
    \]
    \[
    s.t. \quad PX = I
    \]
  - Substitute the solution \( X(P, I) \) back into the utility function to obtain:
    \[
    \text{maximum utility} = V(P, I) = U(X(P, I))
    \]
Example: Cobb-Douglas Utility

- Recall our earlier problem: 
  \[ \max U(X) = x_1^a x_2^b \]
  \[ \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq I \]

- We found: 
  \[ x_1^* = \frac{a}{a+b} \frac{I}{p_1} \]
  \[ x_2^* = \frac{b}{a+b} \frac{I}{p_2} \]

- So \( V(p,I) = \ldots = \frac{I^{a+b}}{p_1^a p_2^b} \left[ \left( \frac{a}{a+b} \right)^a \left( \frac{b}{a+b} \right)^b \right] \)

- What is \( \frac{\partial V(p,I)}{\partial I} \)?
Complements and Substitutes

- Gross substitutes

\[
\frac{\partial x_j(P, I)}{\partial p_i} > 0
\]

- Gross complements

\[
\frac{\partial x_j(P, I)}{\partial p_i} < 0
\]
Demand Elasticity

- Percent change in demand resulting from 1% change in an attribute.

\[
\frac{\text{% change in } X^\ast}{\text{% change in } P} = \frac{\Delta X^\ast / X^\ast}{\Delta P / P}
\]

e.g. price-elasticity

- Own elasticity

\[
\varepsilon_{p_i}^{x_i} = \frac{p_i}{x_i(P, I)} \frac{\partial x_i(P, I)}{\partial p_i}
\]

- Cross elasticity

\[
\varepsilon_{p_i}^{x_j} = \frac{p_i}{x_j(P, I)} \frac{\partial x_j(P, I)}{\partial p_i}
\]
Consumer Surplus (or Welfare)

- Consumer surplus
  - difference between the total value consumers receive from the consumption of a good and the amount paid

\[ \text{Consumer surplus at } p_1^1 \]

Price of good 1

\[ p_1^1 \]

\[ p_1^0 \]

Quantity

\[ x_1(p_1, I) \]

Loss of surplus when price increases from \( p_1^0 \) to \( p_1^1 \)
Consumer Surplus

- Consumer surplus is key for evaluating public policy decisions:
  - Building transportation infrastructure
  - Changing regulations (e.g., emissions)
  - Determining fare and service structures
Review

- Basic concepts
  - Preference, Utility, Rationality

- Other useful details
  - Indirect utility
  - Complements and Substitutes
  - Elasticity
  - Consumer Surplus

Next… Discussion of assumptions
Assumptions

- Impact of consumption of one good on utility of another good?
- Income is the only constraint
- Utility is a function of quantities (a good is a good)
- Demand curves are for an individual
- Behavior is deterministic
- Goods are infinitely divisible (continuous)
Separable Utility

- The consumption of one good does not affect the utility received from some other good

\[ U(X) = \sum_{i=1}^{m} U_i(X_i) \]

- Separability into groups of goods

\[ U(X) = U\left(U_1(X_{g1}), ..., U_k(X_{gk})\right) \]

\[ X_{gk} \] - group \( k \) of goods

- Allows allocating budget in 2 stages:
  - Between groups
  - Within a group
Attributes of Goods

Classic: Only quantities matter.

Quantities $\rightarrow$ Attributes
- The consumer receives utility from attributes of goods rather than the goods themselves
- Example: (dis)utility of an auto trip depends on travel time, cost, comfort etc.

\[
U = U(A) \\
A = A(X)
\]

- $X$ – Goods
- $A$ – Attributes
- $U$ – Utility
“Household Production” Model

- Consumers purchase goods in order to “produce” utility from them, depending on their attributes.
- Classic example: Households purchase food in order to obtain calories and vitamins (could also include enjoyment of taste), which then produce “utility”
Utility of a Transportation Mode

- Consumption bundles: auto, bus, train, etc.
- Utility function
  \[ U_{bus} = \beta_0 + \beta_1 WT_{bus} + \beta_2 TT_{bus} + \beta_3 C_{bus} \]
  - \( WT_{bus} \) – waiting time (minutes)
  - \( TT_{bus} \) – total travel time (minutes)
  - \( C_{bus} \) – total cost of trip (dollars)
- Parameters \( \beta \) represent tastes, and vary by education, gender, trip purpose, etc.
Time Budgets and Value of Time

- Along with *income constraint*, there is also a *time constraint* (e.g., 24 hours in a day)
  -> Gives time *value*.

- Value of time is the marginal rate of substitution between time and cost

\[
U_{bus} = \beta_0 + \beta_1 WT_{bus} + \beta_2 TT_{bus} + \beta_3 C_{bus}
\]

\[
VOT = \frac{MU_{TT}}{MU_C} = \frac{\beta_2}{\beta_3} \frac{\$}{\text{min}}
\]
Aggregate Consumer Demand

- Aggregate demand is the sum of demands of all consumers

\[ X(P, I_1, ..., I_N) = \sum_{n=1}^{N} X_n(P, I_n) \]

- \( n \) – An individual consumer
- \( N \) - # of consumers

- Is this viable? Are individuals sufficiently similar to simply “add up”?
### Heterogeneity

#### Mobility rates, São Paulo, 1987:

<table>
<thead>
<tr>
<th>Family income</th>
<th>Share in population</th>
<th>Mobility rate (all trips)</th>
<th>Mobility rate (motorized trips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;240</td>
<td>20.8%</td>
<td>1.51</td>
<td>0.59</td>
</tr>
<tr>
<td>240-480</td>
<td>28.1%</td>
<td>1.85</td>
<td>0.87</td>
</tr>
<tr>
<td>480-900</td>
<td>26.0%</td>
<td>2.22</td>
<td>1.24</td>
</tr>
<tr>
<td>900-1800</td>
<td>17.2%</td>
<td>2.53</td>
<td>1.65</td>
</tr>
<tr>
<td>&gt;1800</td>
<td>7.9%</td>
<td>3.02</td>
<td>2.28</td>
</tr>
</tbody>
</table>

Introducing Uncertainty

- Random utility model
  \[ U_i = V(\text{attributes of } i; \text{parameters}) + \text{epsilon}_i \]
- Decision maker deterministic, but analyst imperfect due to:
  - Unobserved attributes
  - Unobserved taste variations
  - Measurement errors
  - Use of proxy variables
Continuous vs. Discrete Goods

- Continuous goods
- Discrete goods
Summary

● Basic concepts
  – Preference, Utility, Rationality

● Other useful details
  – Indirect utility
  – Complements and Substitutes
  – Elasticity, Consumer Surplus

● Relaxing the assumptions and working towards practical, empirical models

Next Lecture… Discrete Choice Analysis
Appendix

Dual concepts in demand analysis
Expenditure Function

- The minimal income needed to achieve any level of utility at given prices
- Solution to the problem:

$$E(P,u) = \min PX$$

$$s.t. \quad U(X) \geq u$$

- The dual to the utility maximization problem
Cobb-Douglas Utility

Min \ p_1x_1 + p_2x_2
Subject to \ x_1^a x_2^b \geq u

● Similar formula for the Lagrangean and solve...
● We find:

\[ x_1^* = \left( \frac{p_2 \cdot a}{p_1 \cdot b} \right)^{b/a+b} \ u^{1/a+b} \]

● So:

\[ E(p, u) = p_1x_1^* + p_2x_2^* \]
Consumer Surplus

- Directly related to the expenditure function
- Let $E_1 = E(p_1^1, p_2, ..., p_m, u_0)$
  and $E_0 = E(p_1^0, p_2, ..., p_m, u_0)$

- Change in consumer surplus = $E_0 - E_1$
Compensated Demand Function

- The expenditure problem solution:

\[
E(P,u) = \min PX \quad s.t. \quad U(X) \geq u \quad \Rightarrow \quad X(P,u) = H(P,u)
\]

- \(H(P,u)\) is the compensated demand function
- Shows the demand for a good as a function of prices assuming utility is held constant.
- Substituting back into the objective function:

\[
E(P,u) = \sum_{i} p_i x_i(P,u) = PX(P,u)
\]
Relations Among Demand Concepts

**Primal**
- Max. $U(X)$
  - s.t. $PX = I$
- Indirect utility function
  - $U^* = V(P, I)$
- Demand function
  - $X^* = X(P, I)$

**Dual**
- Min. $E(X)$
  - s.t. $U(X) = u$
- Expenditure function
  - $E^* = E(P, u)$
- Demand function
  - $X^* = H(P, u)$