Airline Revenue Management: Flight Leg and Network Optimization

1.201 Transportation Systems Analysis: Demand & Economics

Dr. Peter P. Belobaba
1. Overview of Airline Pricing
   • Differential Pricing Theory
   • Fare Restrictions and Disutility

2. Revenue Management Systems

3. Overbooking Models

4. Single-leg Fare Class Seat Allocation Problem
   • EMSRb Model for Seat Protection

5. Network Revenue Management
   • Origin-Destination Control Mechanisms
   • Network Optimization Methods
Differential Pricing Theory

- Market segments with different “willingness to pay” for air travel
- Different “fare products” offered to business versus leisure travelers
- Prevent diversion by setting restrictions on lower fare products and limiting seats available
- Increased revenues and higher load factors than any single fare strategy
Traditional Approach: Restrictions on Lower Fares

• Progressively more severe restrictions on low fare products designed to prevent diversion:
  ▪ Lowest fares have advance purchase and minimum stay requirements, as well as cancellation and change fees
  ▪ Restrictions increase the inconvenience or “disutility cost” of low fares to travelers with high WTP, forcing them to pay more
  ▪ Studies show “Saturday night minimum stay” condition to be most effective in keeping business travelers from purchasing low fares

• Still, it is impossible to achieve perfect segmentation:
  ▪ Some travelers with high WTP can meet restrictions
  ▪ Many business travelers often purchase restricted fares
Restrictions Help to Segment Demand

<table>
<thead>
<tr>
<th>Fare Code</th>
<th>Dollar Price</th>
<th>Advance Purchase</th>
<th>Round Trip?</th>
<th>Sat. Night Min. Stay</th>
<th>Percent Non-Refundable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$400</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>$200</td>
<td>7 day</td>
<td>Yes</td>
<td>--</td>
<td>50 %</td>
</tr>
<tr>
<td>M</td>
<td>$150</td>
<td>14 day</td>
<td>Yes</td>
<td>Yes</td>
<td>100 %</td>
</tr>
<tr>
<td>Q</td>
<td>$100</td>
<td>21 day</td>
<td>Yes</td>
<td>Yes</td>
<td>100 %</td>
</tr>
</tbody>
</table>

- Business passengers unwilling to stay over Saturday night will not buy M or Q.
- RM system protects for Y, B demand but keeps M,Q classes open without losing revenue.
Example: Restriction Disutility Costs

Business Passenger Fare Structure, Eb vs. Eb, DF=1
## BOS-SEA Fare Structure
American Airlines, October 1, 2001

<table>
<thead>
<tr>
<th>Roundtrip Fare ($)</th>
<th>Cls</th>
<th>Advance Purchase</th>
<th>Minimum Stay</th>
<th>Change Fee?</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>458</td>
<td>N</td>
<td>21 days</td>
<td>Sat. Night</td>
<td>Yes</td>
<td>Tue/Wed/Sat</td>
</tr>
<tr>
<td>707</td>
<td>M</td>
<td>21 days</td>
<td>Sat. Night</td>
<td>Yes</td>
<td>Tue/Wed</td>
</tr>
<tr>
<td>760</td>
<td>M</td>
<td>21 days</td>
<td>Sat. Night</td>
<td>Yes</td>
<td>Thu-Mon</td>
</tr>
<tr>
<td>927</td>
<td>H</td>
<td>14 days</td>
<td>Sat. Night</td>
<td>Yes</td>
<td>Tue/Wed</td>
</tr>
<tr>
<td>1001</td>
<td>H</td>
<td>14 days</td>
<td>Sat. Night</td>
<td>Yes</td>
<td>Thu-Mon</td>
</tr>
<tr>
<td>2083</td>
<td>B</td>
<td>3 days</td>
<td>none</td>
<td>No</td>
<td>2 X OW Fare</td>
</tr>
<tr>
<td>2262</td>
<td>Y</td>
<td>none</td>
<td>none</td>
<td>No</td>
<td>2 X OW Fare</td>
</tr>
<tr>
<td>2783</td>
<td>F</td>
<td>none</td>
<td>none</td>
<td>No</td>
<td>First Class</td>
</tr>
</tbody>
</table>
Yield Management = Revenue Management

- YM assumes a set of differentiated fare classes and available flight capacity as given:
  - Forecast future booking demand for each fare product
  - Optimize number of seats to be made available to each fare class

- Optimal control of available seat inventory:
  - On high demand flights, limit discount fare and group bookings to increase overall yield (average fare) and revenue.
  - On low demand flights, sell empty seats at any low fare to increase load factors and revenue.
  - Revenue maximization requires a balance of yield and load factor

- Most airlines now refer to “Revenue Management” (RM) instead.
Typical 3rd Generation RM System

- Collects and maintains historical booking data by flight and fare class, for each past departure date.

- Forecasts future booking demand and no-show rates by flight departure date and fare class.

- Calculates limits to maximize total flight revenues:
  - Overbooking levels to minimize costs of spoilage/denied boardings
  - Booking class limits on low-value classes to protect high-fare seats

- Interactive decision support for RM analysts:
  - Can review, accept or reject recommendations
Third Generation RM System

RM Database

- Revenue Data
- Historical Bookings
- No Show Data
- Actual Bookings

Forecasting Models

- Booking Limit Optimization
- Overbooking Model

RM Models

Bookings and Cancellations

Reservations/Inventory System

Booking Limits
Revenue Management Techniques

• Overbooking
  ▪ Accept reservations in excess of aircraft capacity to overcome loss of revenues due to passenger “no-show” effects

• Fare Class Mix (Flight Leg Optimization)
  ▪ Determine revenue-maximizing mix of seats available to each booking (fare) class on each flight departure

• Traffic Flow (O-D) Control (Network Optimization)
  ▪ Further distinguish between seats available to short-haul (one-leg) vs. long-haul (connecting) passengers, to maximize total network revenues
Flight Overbooking

- Determine maximum number of bookings to accept for a given physical capacity.

- Minimize total costs of denied boardings and spoilage (lost revenue).

- U.S. domestic no-show rates can reach 15-20 percent of final pre-departure bookings:
  - On peak holiday days, when high no-shows are least desirable
  - Average no-show rates have dropped, to 10-15% with more fare penalties and better efforts by airlines to firm up bookings

- Effective overbooking can generate as much revenue gain as fare class seat allocation.
Cost-Based Overbooking Model

• Find AU that minimizes:

\[ \text{Total Cost} = DB \times E[DB] + SP \times E[SP] \]

$DB$ and $SP$ = cost per DB and SP, respectively

$E[DB] = \text{expected number of DBs, given AU}$

$E[SP] = \text{expected number of SP seats, given AU}$

• For any given AU:

• Mathematical search over range of AU values to find minimum total cost.
Cost-Based Overbooking Model

Minimize total cost of expected Denied Boardings plus Spoiled Seats

Optimal AU = 123
2007 US Involuntary DBs per 10,000
Flight Leg Revenue Maximization

• **Given for a future flight leg departure date:**
  - Total remaining booking capacity of (typically) the coach compartment
  - Several fare (booking) classes that share the same inventory of seats in the compartment
  - Forecasts of future booking demand by fare class between current DCP and departure
  - Revenue estimates for each fare (booking) class

• **Objective is to maximize total expected revenue:**
  - Protect seats for each fare class based on revenue value, taking into account forecast uncertainty and probability of realizing the forecasted demand
Serially Nested Buckets

BL1 = Cap

Protected for class 1 from class 2, 3, ..., N

Protected for classes 1 and 2 from class 3, 4, ..., N

BL2

BL3
EMSRb Model for Seat Protection: Assumptions

- **Modeling assumptions for serially nested classes:**
  a) demand for each class is separate and independent of demand in other classes.
  b) demand for each class is stochastic and can be represented by a probability distribution.
  c) lowest class books first, in its entirety, followed by the next lowest class, etc.
  d) booking limits are only determined once (i.e., static optimization model)

- **Problem is to find protection levels for higher classes, and booking limits on lower classes**
EMSرب Model Calculations

- To calculate the optimal protection levels:
  Define \( P_i(S_i) \) = probability that \( X_i \geq S_i \),
  where \( S_i \) is the number of seats made available to class i, \( X_i \) is
  the random demand for class i

- The expected marginal revenue of making the Sth seat available to class i is:
  \( \text{EMSR}_i(S_i) = R_i \times P_i(S_i) \) where \( R_i \) is the average revenue (or fare)
  from class i

- The optimal protection level, \( \pi_1 \) for class 1 from class 2 satisfies:
  \( \text{EMSR}_1(\pi_1) = R_1 \times P_1(\pi_1) = R_2 \)
Example Calculation

Consider the following flight leg example:

<table>
<thead>
<tr>
<th>Class</th>
<th>Mean Fcst.</th>
<th>Std. Dev.</th>
<th>Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>10</td>
<td>3</td>
<td>1000</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>5</td>
<td>700</td>
</tr>
<tr>
<td>M</td>
<td>20</td>
<td>7</td>
<td>500</td>
</tr>
<tr>
<td>Q</td>
<td>30</td>
<td>10</td>
<td>350</td>
</tr>
</tbody>
</table>

To find the protection for the Y fare class, we want to find the largest value of $\pi_Y$ for which $\text{EMSR}_Y(\pi_Y) = R_Y \cdot P_Y(\pi_Y) \geq R_B$
Example (cont’d)

$\text{EMSR}_Y(\pi_Y) = 1000 \times P_Y(\pi_Y) \geq 700$

$P_Y(\pi_Y) \geq 0.70$

where $P_Y(\pi_Y) =$ probability that $X_Y \geq \pi_Y$.

- Assume demand in Y class is *normally* distributed, then we can create a standardized normal random variable as $(X_Y - 10)/3$:
  - for $\pi_Y = 7$, $\text{Prob} \{ (X_Y - 10)/3 \geq (7 - 10)/3 \} = 0.841$
  - for $\pi_Y = 8$, $\text{Prob} \{ (X_Y - 10)/3 \geq (8 - 10)/3 \} = 0.747$
  - for $\pi_Y = 9$, $\text{Prob} \{ (X_Y - 10)/3 \geq (9 - 10)/3 \} = 0.63$

- $\pi_Y = 8$ is the largest integer value of $\pi_Y$ that gives a probability $\geq 0.7$ and we will protect 8 seats for Y class.
General Case for Class n

- Joint protection for classes 1 through n from class n+1

\[
\overline{X}_{1,n} = \sum_{i=1}^{n} \overline{X}_i
\]

\[
\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^{n} \hat{\sigma}_i^2}
\]

\[
R_{1,n} = \frac{\sum_{i=1}^{n} R_i \ast \overline{X}_i}{\overline{X}_{1,n}}
\]

- We then find the value of \( \pi_n \) that makes

\[
\text{EMSR}_{1,n}(\pi_n) = R_{1,n} \ast P_{1,n}(\pi_n) = R_{n+1}
\]

- Once \( \pi_n \) is found, set BL_{n+1} = Capacity - \( \pi_n \)
EMSRb Seat Protection Model

<table>
<thead>
<tr>
<th>CLASS</th>
<th>AVERAGE FARE</th>
<th>AVAILABLE SEATS</th>
<th>BOOKED MEAN</th>
<th>SIGMA PROTECT</th>
<th>joINT BOOKING LIMIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$670</td>
<td>135</td>
<td>0</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>M</td>
<td>$550</td>
<td>135</td>
<td>0</td>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>$420</td>
<td>135</td>
<td>0</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>V</td>
<td>$310</td>
<td>135</td>
<td>0</td>
<td>22</td>
<td>9</td>
</tr>
<tr>
<td>Q</td>
<td>$220</td>
<td>135</td>
<td>0</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>L</td>
<td>$140</td>
<td>135</td>
<td>0</td>
<td>47</td>
<td>14</td>
</tr>
<tr>
<td>SUM</td>
<td></td>
<td>0</td>
<td>135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dynamic Revision and Intervention

- **RM systems revise forecasts and re-optimize booking limits at numerous “checkpoints”:**
  - Monitor actual bookings vs. previously forecasted demand
  - Re-forecast demand and re-optimize at fixed checkpoints or when unexpected booking activity occurs
  - Can mean substantial changes in fare class availability from one day to the next, even for the same flight departure

- **Substantial proportion of fare mix revenue gain comes from dynamic revision of booking limits:**
  - Human intervention is important in unusual circumstances, such as “unexplained” surges in demand due to special events
Revision of Forecasts and Limits as Bookings Accepted

<table>
<thead>
<tr>
<th>CABIN CAPACITY =</th>
<th>135</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVAILABLE SEATS  =</td>
<td>63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BOOKING CLASS</th>
<th>AVERAGE FARE</th>
<th>SEATS BOOKED</th>
<th>FORECAST DEMAND</th>
<th>JOINT PROTECT LIMIT</th>
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</thead>
<tbody>
<tr>
<td>Y</td>
<td>$670</td>
<td>2</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>M</td>
<td>$550</td>
<td>4</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>$420</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>V</td>
<td>$310</td>
<td>12</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Q</td>
<td>$220</td>
<td>17</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>L</td>
<td>$140</td>
<td>32</td>
<td>15</td>
<td>4</td>
</tr>
</tbody>
</table>

| SUM           | 72           | 73           |

*Higher than expected Q bookings close L class*
Network RM: O-D Control

- Advanced airlines are developing O-D control after having mastered basic leg/class RM controls
  - Effective leg-based fare class control and overbooking alone can increase total system revenues by 4 to 6%
- “The capability to respond to different O-D requests with different seat availability.”
- Effective O-D control can further increase total network revenues by 1 to 2%
  - Depends on network structure and connecting flows
  - O-D control gains increase with average load factor
  - But implementation is more difficult than leg-based RM systems
O-D Control Example: Hub Network

- BOS-NRT $800 (Deep Discount Q Fare)
- BOS-ATL $500 (Full Y Fare)
- BOS-MEX $650 (Discount M Fare)
- ATL-MEX $400 (Full Y Fare)
Marginal Value of Last Seat on a Leg

• Marginal value concept is basis of leg RM:
  ▪ Accept booking in fare class if revenue value exceeds marginal value of last (lowest valued) remaining available seat on the flight leg

• In network RM, need to estimate marginal network value of last seat on each leg:
  ▪ Can be used as “displacement cost” of a connecting vs. local passenger
  ▪ Or, as a minimum acceptable “bid price” for the next booking on each leg
Marginal Network Value of Last Seat

EMSR($) vs Seats

ODF #1

ODF #1,2

ODF #1,2,3

EMSRc

Available Seats

29
Displacement Cost Concept

• Contribution of an ODF to network revenue on a leg is less than or equal to its total fare:
  ▪ Connecting passengers can displace revenue on down-line (or up-line) legs

• Given estimated down-line displacement, ODFs are mapped based on network value:
  ▪ Network value on Leg 1 = Total fare minus sum of down-line leg displacement costs
  ▪ Under high demand, availability for connecting passengers is reduced, locals get more seats
Virtual Class Mapping with Displacement

FARE VALUES BY ITINERARY

<table>
<thead>
<tr>
<th>NCE/FRA</th>
<th>FARE (OW)</th>
<th>NCE/HKG (via FRA)</th>
<th>NCE/JFK (via FRA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLASS</td>
<td>FARE (OW)</td>
<td>CLASS</td>
<td>FARE (OW)</td>
</tr>
<tr>
<td>Y</td>
<td>$450</td>
<td>Y</td>
<td>$1415</td>
</tr>
<tr>
<td>B</td>
<td>$380</td>
<td>B</td>
<td>$975</td>
</tr>
<tr>
<td>M</td>
<td>$225</td>
<td>M</td>
<td>$770</td>
</tr>
<tr>
<td>Q</td>
<td>$165</td>
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<td>$590</td>
</tr>
<tr>
<td>V</td>
<td>$135</td>
<td>V</td>
<td>$499</td>
</tr>
</tbody>
</table>

MAPPING OF ODFs ON NCE/FRA LEG TO VIRTUAL VALUE CLASSES

<table>
<thead>
<tr>
<th>VIRTUAL CLASS</th>
<th>REVENUE RANGE</th>
<th>MAPPING OF O-D MARKETS/CLASSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1200 +</td>
<td>Y NCEHKG</td>
</tr>
<tr>
<td>2</td>
<td>900-1199</td>
<td>B NCEHKG Y NCEJFK</td>
</tr>
<tr>
<td>3</td>
<td>750-899</td>
<td>M NCEHKG</td>
</tr>
<tr>
<td>4</td>
<td>600-749</td>
<td>B NCEJFK</td>
</tr>
<tr>
<td>5</td>
<td>500-599</td>
<td>Q NCEHKG M NCEJFK</td>
</tr>
<tr>
<td>6</td>
<td>430-499</td>
<td>V NCEHKG Y NCEFRA</td>
</tr>
<tr>
<td>7</td>
<td>340-429</td>
<td>B NCEFRA Q NCEJFK</td>
</tr>
<tr>
<td>8</td>
<td>200-339</td>
<td>V NCEJFK M NCEFRA</td>
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<tr>
<td>9</td>
<td>150-199</td>
<td>Q NCEFRA</td>
</tr>
<tr>
<td>10</td>
<td>0 - 149</td>
<td>V NCEFRA</td>
</tr>
</tbody>
</table>

Displacement Adjustment
Bid Price Concept

• Marginal value of last seat can also represent the flight leg “Bid Price”:
  ▪ A minimum “cutoff” value required to accept a booking request
  ▪ For a single-leg itinerary, a request is accepted if the corresponding fare is greater than the bid price for the leg.
  ▪ For a multi-leg itinerary, the ODF fare must be greater than the sum of the bid prices of all flight legs used by the itinerary.

• Much simpler inventory control mechanism than virtual buckets:
  ▪ Simply need to store bid price value for each leg
  ▪ Must revise bid prices frequently to prevent too many bookings of ODFs at current bid price
Example: Bid Price Control

A -----> B -----> C -----> D

• Given leg bid prices
  A-B: $34  B-C: $201  C-D: $169

• Availability for O-D requests B-C:

<table>
<thead>
<tr>
<th>Bid Price</th>
<th>Available?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$440</td>
<td>Yes</td>
</tr>
<tr>
<td>$315</td>
<td>Yes</td>
</tr>
<tr>
<td>$223</td>
<td>Yes</td>
</tr>
<tr>
<td>$197</td>
<td>No</td>
</tr>
</tbody>
</table>
### A-B: $34  B-C: $201  C-D: $169

<table>
<thead>
<tr>
<th>A-C</th>
<th>Bid Price</th>
<th>Available?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$519</td>
<td>Yes</td>
</tr>
<tr>
<td>M</td>
<td>$344</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>$262</td>
<td>Yes</td>
</tr>
<tr>
<td>Q</td>
<td>$231</td>
<td>No</td>
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</table>

<table>
<thead>
<tr>
<th>A-D</th>
<th>Bid Price</th>
<th>Available?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>$582</td>
<td>Yes</td>
</tr>
<tr>
<td>M</td>
<td>$379</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>$302</td>
<td>No</td>
</tr>
<tr>
<td>Q</td>
<td>$269</td>
<td>No</td>
</tr>
</tbody>
</table>
Network Optimization Methods

- Network optimization mathematics needed for both bid price and value bucket controls.

- Several optimization methods to consider:
  - Deterministic Linear Programming
  - Dynamic Programming
  - Nested Probabilistic Network Bid Price

- Simulated revenue gains are quite similar:
  - ODF database, forecast accuracy and robustness under realistic conditions make a bigger difference
Network Linear Program (LP)

Maximize Total Revenue = Sum [Fare * Seats]

- Summed over all ODFs on network

Subject to following constraints:

- Seats for each ODF <= Mean Forecast Demand
- Sum[Seats on Each Leg] <= Leg Capacity

Outputs of LP solution:

- Seats allocated to each ODF (not useful)
- "Shadow price" on each leg (reflects network revenue value of last seat on each flight leg)
### O-D Control System Alternatives

<table>
<thead>
<tr>
<th>O-D Control System</th>
<th>Data and Forecasts</th>
<th>Optimization Model</th>
<th>Control Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rev. Value Buckets</td>
<td>Leg/bucket</td>
<td>Leg EMSR</td>
<td>Leg/bucket Limits</td>
</tr>
<tr>
<td>Heuristic Bid Price</td>
<td>Leg/bucket</td>
<td>Leg EMSR</td>
<td>Bid Price for Connex only</td>
</tr>
<tr>
<td>Disp. Adjust. Value Bkts.</td>
<td>ODF</td>
<td>Network + Leg EMSR</td>
<td>Leg/bucket Limits</td>
</tr>
<tr>
<td>Network Bid Price</td>
<td>ODF</td>
<td>Network</td>
<td>O-D Bid Prices</td>
</tr>
</tbody>
</table>
O-D Revenue Gain Comparison
Airline A, O-D Control vs. Leg/Class RM

Network Load Factor

- 70%
- 78%
- 83%
- 87%

<table>
<thead>
<tr>
<th>Network Load Factor</th>
<th>HBP</th>
<th>DAVN</th>
<th>PROBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>78%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>83%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>87%</td>
<td></td>
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</table>
Summary: Airline O-D RM Systems

• O-D control is the 4th generation of RM:
  ▪ Data collection, forecasting, optimization and control by origin-destination-fare type as well as distribution channel

• Provides control by itinerary and network value of requests, not simply by flight leg and class
  ▪ Incremental network revenue gains of 1-2% over basic RM
  ▪ Essential to protect against revenue loss to competitors
  ▪ Increased control of valuable inventory in the face of pricing pressures, new distribution channels, and strategic alliances