Real-Time Control Strategies for Rail Transit

Outline:

• Problem Description and Motivation
• Model Formulation
• Model Application and Results
• Implementation Issues
• Conclusions
Problem Context

• High frequency urban rail service (e.g. headways of 2-10 minutes)
  – passengers arrive randomly
  – service regularity is a key goal
  – \( E(WT) = \frac{E(H) [1 + \text{cov}(H)^2]}{2} \)

• Branching route structure

• Central real-time train location information and dispatch capability
Three Levels of Control Problems

**Routine disturbances** - several minutes’ deviation from schedule

Control Strategies:
- speed adjustment
- dwell time adjustment (selective holding) terminal recovery

**Short-term disruptions**: 5-30 minute blockages on the line

**Longer-term disruptions** - greater than 30 minute blockages

Control Strategies:
- single-track reverse direction operations
- replacement bus service around blockage
Disruption Response Strategies

Terminal A

short-turn option 1

Express train after clearance

Blockage

short-turn option 2

Held trains

Terminal B

KEY
- Station
- Train
Problem Description

- **Overall Objective:**
  -- Develop a real-time decision support system to determine control strategies to recover from disruptions

- **Specific Objective:**
  -- Minimize passenger waiting times (implies maintaining even headways)

- **Key Characteristics:**
  -- Instability of even headways
  -- Passenger sensitivity to long waiting time and crowding
  -- Cost insensitivity to different strategies

- **Possible Strategies:**
  -- Holding
  -- Short-turning
  -- Expressing
Example of Transit Control Strategies

- 6-minute scheduled headways
- 3-minute minimum safe headway
- 10-minute disruption
- Impact set includes trains T2, T3, and T4 and stations S1 and S2
Example Results

1. **Do nothing:**  \( h_{T2} = 6 \text{ mins.}; \ h_{T3} = 16 \text{ mins.}; \ h_{T4} = 3 \text{ mins.} \)

   \[
   \text{Total Passenger Waiting Time} = \frac{1}{2} \left[ 4(16^2 + 3^2) \right] + \frac{1}{2} \left[ 20(6^2 + 16^2 + 3^2) \right] = 3540 \text{ pass – mins.}
   \]

2. **Holding:**  Hold T2 at S2 for 4 mins.
   Then at S2: \( h_{T2} = 10 \text{ mins.}; \ h_{T3} = 12 \text{ mins.}; \ h_{T4} = 3 \text{ mins.} \)

   \[
   TPWT = \frac{1}{2} \left[ (4(16^2 + 3^2) + 20(10^2 + 12^2 + 3^2)) \right] = 3060 \text{ pass – mins.}
   \]

3. **Expressing:**  Express T3 past S1 to save 1 minute in travel time.
   Then at S2: \( h_{T2} = 6 \text{ mins.}; \ h_{T3} = 15 \text{ mins.}; \ h_{T4} = 4 \text{ mins.} \)

   \[
   TPWT = \frac{1}{2} \left[ 4 \ast 19^2 + 20(6^2 + 15^2 + 4^2) \right] = 3492 \text{ pass – mins.}
   \]
Model Formulation

Key Features:

- station specific parameters: passenger arrival rates, alighting fractions, minimum safe headways
- station dwell time a linear function of passengers boarding, alighting and crowding
- train order is variable
- train capacity constraint

Simplifications:

- predictable disruption length
- passenger flows estimated from historical data
- system is modelled as deterministic
- strategies selected to produce minimum inter-station travel times.
Model Formulation

Decision Variables: departure time of train $i$ from station $k$

Objective function: minimization of passenger waiting time
- quadratic function approximated by a piecewise linear function

Impact Set: consider a finite set of trains and stations and approximate the effects beyond this set

Constraints: train running time and minimum safe headways
- other relationships govern passenger loads, train dwell times

Model Structure: mixed integer program except if passenger capacity is not binding when it is a linear program
Specific Models

Holding Strategy Models:
- Hold all
- Hold once
- Hold at first station

Combined Short-turning and Holding Models:
- Predetermined train order
- Undetermined train order
Model Application

MBTA Red Line Characteristics:
• 23 stations (including 3 terminals)
• 27 six-car trains in A.M. peak
• 3.4 minute trunk headways (6 and 8 minutes on branches)
• 30,000 passengers in peak hour
Red Line

Blockage Location, Incident 1

Blockage Location, Incident 2

Park Street

Harvard Square

Alewife

Ashmont

Braintree

Harvard Square

Kendall/MIT

Blockage Location, Incident 1

KEY:
- △ Ashmont Train
- ● Braintree Train
- Station
- Blockage

12/08/03

1.224J/ESD.204J
## Incident 1, Ten Minute Delay

<table>
<thead>
<tr>
<th>Control Strategies</th>
<th>FOHPC</th>
<th>STPP</th>
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<tbody>
<tr>
<td>Passenger Waiting Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Passenger-Minutes)</td>
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<td></td>
</tr>
<tr>
<td>Do Nothing</td>
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<td></td>
</tr>
<tr>
<td>Hold All</td>
<td>8863</td>
<td>9997</td>
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<tr>
<td>Hold Once</td>
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<td>Savings (percent)</td>
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<td>Behind Blockage</td>
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<tr>
<td>Savings (percent)</td>
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<td>0%</td>
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<tr>
<td>Maximum Train Load</td>
<td>988</td>
<td>603</td>
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<tr>
<td>Problem Size</td>
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<td>88</td>
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<tr>
<td>CPU Time (seconds)</td>
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<td>16</td>
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# Incident 1, Twenty Minute Delay

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## Incident 2, Ten Minute Delay

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<td>Hold Once</td>
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### Incident 2, Twenty Minute Delay

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<td>Problem Size</td>
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Impact Set Size

Incident 1, Twenty Minute Delay

Number of Trains Held

Passenger Waiting Time Saved (minutes)
## Passenger On-Board Time

<table>
<thead>
<tr>
<th>Incident</th>
<th>Delay</th>
<th>Objective Function</th>
<th>Passenger Time</th>
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<tr>
<td></td>
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<td></td>
<td>Waiting</td>
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<tr>
<td>1</td>
<td>10 Min.</td>
<td>PWT</td>
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<tr>
<td></td>
<td></td>
<td>TPT</td>
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<tr>
<td>1</td>
<td>20 Min.</td>
<td>PWT</td>
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<td>2</td>
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<td>50018</td>
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<td>TPT</td>
<td>51201</td>
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Execution Times

- Sun SPARC 20 workstation
- GAMS V. 2.25
- CPLEX V. 3.0
- Simple front-end heuristic to fix some binary variables

Large Problems: 11-13 trains, 69-95 train/station decision var.
Execution Time: 10 out of 16 <30 sec.

Realistic Size: 7-8 trains, 40-50 train/station decision var.
Execution Time: 16 out of 16 <34 sec.
Conclusions

• Holding and short-turning models formulated and solved to optimality
• Active control strategies result in significant passenger waiting time savings
• Train control set can be reduced to trains ahead of the blockage
• Train control set need not be large
Conclusions

• Hold at First or Hold Once strategies can be almost as effective as Hold All strategy

• Short-turning most effective where:
  -- blockage is long relative to short-turn time
  -- number of stations outside the short-turn loop is small

• Consideration of on-board time is desirable

• Execution time is 30 seconds or less but faster heuristics are probably achievable