Outline

• Real Time Control Strategies for Rail Transit
  • Prior Research
  • Shen/Wilson Model Formulation
  • Model Application and Results
  • Implementation Issues
  • Conclusions

• Follow-on Subjects

• Final Exam
Prior Research

O’Dell and Wilson (1999):

• Formulated and solved to optimality holding and (restricted) short-turning models
• Active control strategies resulted in significant passenger wait time savings
• Train impact set need not be large and can be restricted to trains ahead of the blockage
• Hold-at-first station strategy is recommended
• Short-turning is most effective where:
  -- blockage is long relative to short-turn time
  -- number of stations outside short-turn loop is small
• Solution time is typically 30 seconds or less
Prior Research

Limitations:

• only specified short-turns included in solution
• expressing not included
• objective function ignored in-vehicle delay time
• did not recognize the stochastic nature of disruption duration
Model Formulation

Key Features:

- station specific parameters: passenger arrival rates, alighting fractions, minimum safe headways
- station dwell time a linear function of passengers boarding, alighting and crowding
- train order is variable
- train capacity constraint

Simplifications:

- predictable disruption length
- passenger flows estimated from historical data
- system is modelled as deterministic
- strategies selected to produce minimum inter-station travel times.
Shen/Wilson Model Formulation*

Decision variables:
- departure time of train \( i \) from station \( k \)
- short-turning binary variables
- expressing binary variables

Objective function:
- minimization of weighted sum of passenger waiting time at stations and in-vehicle delay

Control set:
- set of trains and stations where control actions may be applied, typically:
  -- 2-4 holding candidates ahead of the disruption
  -- 1-2 expressing candidates behind the disruption
  -- 1-3 short-turning candidates

Model Formulation

Impact set:
- consider a finite set of trains and stations over which to evaluate the impacts of the control strategies

Constraints include:
- train running time and minimum safe separation
- train dwell time = \( f \) (passengers boarding and alighting)
- passenger loads and train capacity

Model Structure:
- mixed integer program
Model Simplifications

A. Piece-wise linear approximation of quadratic terms in objective function:
   • waiting time
   • holding time

B. Simplification of non-separable terms
   • additional waiting time for passengers left behind:
     -- approximate headway by minimum headway
   • in-vehicle delay:
     -- approximate passengers on train by normal passenger load at that time and point on route
Model Applications

MBTA Red Line Characteristics:

- 23 stations (including 3 terminals)
- 27 six-car trains in A.M. peak
- 3.4 minute trunk headways (6 and 8 minutes on branches)
- 30,000 passengers in peak hour

Simplified system:

- single loop
- scaled passenger arrival rates and minimum safe separation on trunk portion of line
- 6-minute headways
Scenario Description

KEY:
- △ Ashmont Train
- • Braintree Train
- □ Station
- ★ Blockage

Ashmont
Braintree
Harvard Square
Kendall/MIT
Alewife
Park Street
Incident
North

10 December 2003
10 December 2003
1.224J/ESD.204J
Lecture 13
### Comparison of Strategy Effectiveness for 10-Minute Disruption Scenario

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Mean Platform Waiting Time (min)</th>
<th>Mean In-Vehicle Delay (min)</th>
<th>Mean Weighted Waiting Time (min)</th>
<th>Saving over NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND</td>
<td>3.00</td>
<td>0.00</td>
<td>3.00</td>
<td>-</td>
</tr>
<tr>
<td>NC</td>
<td>5.70</td>
<td>0.15</td>
<td>5.78</td>
<td>-</td>
</tr>
<tr>
<td>H</td>
<td>4.53</td>
<td>1.39</td>
<td>5.23</td>
<td>10%</td>
</tr>
<tr>
<td>HE</td>
<td>4.59</td>
<td>0.83</td>
<td>5.00</td>
<td>13%</td>
</tr>
<tr>
<td>HET</td>
<td>3.55</td>
<td>0.39</td>
<td>3.74</td>
<td>35%</td>
</tr>
</tbody>
</table>

ND = No Disruption  
NC = No Control  
H = Holding Only  
HE = Holding and Expressing Only  
HET = Holding, Expressing, and Short-Turning
## Comparison of Strategy Effectiveness for 20-Minute Disruption Scenario

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Mean Platform Waiting Time (min)</th>
<th>Mean In-vehicle Delay (min)</th>
<th>Mean Weighted Waiting Time (min)</th>
<th>Saving over NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>9.11</td>
<td>0.19</td>
<td>9.20</td>
<td>-</td>
</tr>
<tr>
<td>H</td>
<td>6.57</td>
<td>1.98</td>
<td>7.56</td>
<td>18%</td>
</tr>
<tr>
<td>HE</td>
<td>6.23</td>
<td>1.75</td>
<td>7.10</td>
<td>23%</td>
</tr>
<tr>
<td>HET</td>
<td>3.79</td>
<td>0.35</td>
<td>3.97</td>
<td>57%</td>
</tr>
</tbody>
</table>

NC = No Control    H = Holding Only    HE = Holding and Expressing Only
HET = Holding, Expressing, and Short-Turning
Sensitivity Analysis: Effect of Under-estimating Disruption Duration

<table>
<thead>
<tr>
<th>Blockage Duration Estimate</th>
<th>15 Minutes</th>
<th>10 Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Schemes</td>
<td>H</td>
<td>HE</td>
</tr>
<tr>
<td>Mean Weighted Waiting Time (min)</td>
<td>6.34</td>
<td>5.97</td>
</tr>
<tr>
<td>Increase due to Inaccurate Estimate</td>
<td>+0.5%</td>
<td>+4.0%</td>
</tr>
</tbody>
</table>

H = Holding Only HE = Holding and Expressing Only HET = Holding, Expressing, and Short-Turning
## Sensitivity Analysis: Effect of Over-estimating the Disruption Duration

<table>
<thead>
<tr>
<th>Blockage Duration Estimate</th>
<th>5 Minutes</th>
<th>10 Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Schemes</td>
<td>H, HE, HET</td>
<td>H &amp; HE</td>
</tr>
<tr>
<td>Total Weighted Waiting Time (min)</td>
<td>14875</td>
<td>14888</td>
</tr>
<tr>
<td>Increase due to Wrong Estimate</td>
<td>-</td>
<td>+0.6%</td>
</tr>
</tbody>
</table>

H = Holding Only  
HE = Holding and Expressing Only  
HET = Holding, Expressing, and Short-Turning
Solution Times

- Micron P-II, 300 MHz, 64 MB RAM computer
- C-PLEX v. 4.0

<table>
<thead>
<tr>
<th>Scenario</th>
<th>H</th>
<th>HE</th>
<th>HET</th>
<th>HT</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-Minute</td>
<td>2.91</td>
<td>5.60</td>
<td>11.28</td>
<td>12.06</td>
</tr>
<tr>
<td>20-Minute</td>
<td>12.10</td>
<td>155.01</td>
<td>68.32</td>
<td>24.72</td>
</tr>
</tbody>
</table>

H = Holding Only
HE = Holding and Expressing Only
HET = Holding, Expressing, and Short-Turning
HT = Holding and Short-Turning Only
Conclusions

- Holding provides 10-18% passenger waiting time savings over the no-control case
- Expressing provides little incremental benefit over holding
- Short-turning combined with holding can provide substantial savings: in the case analyzed, 35-57% savings.
- Holding is not sensitive to errors in estimating disruption deviation, but short-turning can be
- Solution time is typically less than 30 seconds
Future Directions

• Develop robust disruption control models recognizing key stochastic elements such as disruption duration, running time, dwell time, and passenger loads

• Develop fast routine control models incorporating control strategies such as speed variation and dwell time variation
Follow-on Subjects

- Optimization
  - 15.057 Systems Optimization
  - 15.093 Optimization Methods
  - 15.094 Systems Optimization: Models and Computation
  - 15.081 Introduction to Mathematical Programming
  - 15.082 Network Optimization
  - 15.083 Combinatorial Optimization
  - 15.084 Nonlinear Programming

- Transportation and Logistics/Optimization
  - 1.206J/16.77J Airline Schedule Planning
  - 1.258J/11.541J/ESD.226J Public Transportation Service and Operations Planning
  - 1.270J/ESD.270J Logistics and Supply Chain Management
Final Exam

- Tuesday, December 16, Room 4-149, 9 AM - noon
- 1 8.5x11" page of notes, both sides
- Focus on modeling and basics of optimization