1.224J/ ESD.204J: Airline Crew Scheduling

Outline

– Crew Scheduling
  • Problem Definition
    – Costs
  • Set Partitioning Model and Solution
– Enhanced Crew Scheduling
  • Link to Maintenance Routing Problem
  • Enhanced Model and Solution Approach
Airline Schedule Planning

Schedule Design

Fleet Assignment

Aircraft Routing

Crew Scheduling

Select optimal set of *flight legs* in a schedule

A flight specifies origin, destination, and departure time

Route individual aircraft honoring maintenance restrictions

Assign crew (pilots and/or flight attendants) to flight legs
Crew Scheduling: Some Background

• This problem has been studied by operations researchers for at least 4 decades

• Most major U.S. airlines use crew pairing optimizers for the cockpit crews
  – Crew costs are the airlines’ second largest operating expense
  – Even small improvements in efficiency can have large financial benefits
Airline Crew Scheduling

• 2-stage process:
  – Crew Pairing Optimization
    • Construct minimum cost work schedules, called pairings, spanning several days
  – Bidline Generation/ Rostering
    • Construct monthly work schedules from the pairings generated in the first stage
      – Bidlines
      – Individualized schedules
    • Objective to balance workload, maximize number of crew requests granted, etc.
Some Definitions

• A crewbase is the home station, or domicile, of a crew

• A crew pairing is a sequence of flights that can be flown by a single crew:
  – beginning and ending at a crewbase
  – spanning one or more days
  – satisfying FAA rules and collective bargaining agreements, such as:
    • maximum flying time in a day
    • minimum rest requirements
    • minimum connection time between two flights
Example: A Crew Pairing

LA
Detroit
Boston

a
b
c
d
e
f
Some More Definitions

• A *duty period* (or *duty*) is a sequence of flight legs comprising a day of work for a crew
  – Alternative pairing definition: a *crew pairing* is a sequence of *duties* separated by rests

• A *crew schedule* is a sequence of *pairings* separated by time-off, satisfying numerous restrictions from regulatory agencies and collective bargaining agreements
Example: Duty Periods

Pairing = DP1(a,b,c) + rest + DP2(d,e) + rest + DP3(f)
Crew Pairing Problem (CP)

• Assign crews to flights such that every flight is covered, costs are minimized and labor rules are satisfied:
  – Maximum flying time in a day
  – Minimum rest requirements
  – Minimum connection time
Crew Pairing Costs

• Duty costs: Maximum of 3 elements:
  – $f1*$flying time cost
  – $f2*$elapsed time cost
  – $f3*$minimum guarantee

• Pairing costs: Maximum of 3 elements:
  – $f1*$duty cost
  – $f2*$time-away-from-base
  – $f3*$minimum guarantee
Set Partitioning Model for CP: Variable Definition and Constraints

• A variable is a *pairing*
  – Binary variables: \( = 1 \) if pairing is assigned to a crew; \( = 0 \) if pairing not flown

• Set partitioning constraints require each flight to be covered exactly once

• Number of possible pairings (variables) grows exponentially with the number of flights
An Example

Flights:
A B C D E F G H

Potential pairings:
- A-C-D-F (y₁): $1
- A-B-E-F (y₂): $2
- C-D-G-H (y₃): $4
- B-E-G-H (y₄): $6

Crew pairing solutions:
- x₁ => pairings 1, 4: $7
- x₂ => pairings 2, 3: $6
Pairing 1

A → C → D → F

B → E → G → H
Pairing 2

A → C → D → F
B → E → G → H
Pairing 3

A → C → D → F

B → E → G → H
Pairing 4

A → C → D → F
B → E → G → H
Notation

- $P^k$ is the set of feasible pairings for fleet family $k$
- $F^k$ is the set of daily flights assigned to fleet family $k$
- $\delta_{fp}$ equals 1 if flight $f$ is in pairing $p$, else 0
- $c_p$ is the cost of pairing $p$
- $y_p$ is 1 if pairing $p$ is in the solution, else 0
Formulation

\[
\begin{align*}
\min & \sum_{p \in P^k} c_p y_p \\
\text{st} & \sum_{p \in P^k} \delta_{fp} y_p = 1 \quad \forall f \in F^k \\
y_p & \in \{0,1\} \quad \forall p \in P^k
\end{align*}
\]
### Flight Cover Constraints:

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1 + y_2 = 1$</td>
<td>A</td>
</tr>
<tr>
<td>$y_2 + y_4 = 1$</td>
<td>B</td>
</tr>
<tr>
<td>$y_1 + y_3 = 1$</td>
<td>C</td>
</tr>
<tr>
<td>$y_1 + y_3 = 1$</td>
<td>D</td>
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<tr>
<td>$y_2 + y_4 = 1$</td>
<td>E</td>
</tr>
<tr>
<td>$y_1 + y_2 = 1$</td>
<td>F</td>
</tr>
<tr>
<td>$y_3 + y_4 = 1$</td>
<td>G</td>
</tr>
<tr>
<td>$y_3 + y_4 = 1$</td>
<td>H</td>
</tr>
</tbody>
</table>

Binary Pairing Restrictions:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>∈ {0,1}</td>
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<tr>
<td>$y_2$</td>
<td>∈ {0,1}</td>
</tr>
<tr>
<td>$y_3$</td>
<td>∈ {0,1}</td>
</tr>
<tr>
<td>$y_4$</td>
<td>∈ {0,1}</td>
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</tbody>
</table>
Set Partitioning Model: Advantages and Disadvantages

• Advantages:
  – Easy to model complex work rules
    • Very few constraints
    • Linear objective function and constraints

• Disadvantages:
  – Huge number of variables—number of variables grows exponentially with the number of flights
Problem Size

- A typical US airline (with a hub-and-spoke network) has millions or billions of potential pairings
  - Example
    - 150 flights 90,000 pairings
    - 250 flights 6,000,000 pairings
- Need a specialized approach to consider problems of this size
Branch-and-Price: Branch-and-Bound for Large-Scale Integer Programs

All possible solutions at leaf nodes of tree ($2^n$ solutions, where $n$ is the number of variables)
Column Generation

Millions/Billions of Variables

Constraints

Initial  Added  Never Considered
LP Solution: Column Generation

• Step 1: Solve Restricted Master Problem
• Step 2: Solve Pricing Problem (generate columns with negative reduced cost)
• Step 3: If columns generated, return to Step 1; otherwise STOP
Network Representation

City A

City B

City C

City D

800 1200 1600 2000

800 1200 1600 2000
Branch-and-Bound with Too Many Variables

- Branch-and-Price
  - Branch-and-bound with bounding provided by LP solutions
  - CP has too many variables to consider all of them
- Solve linear programming relaxation using column generation
A Twist…

• Crew scheduling is critical to the airline industry
  – Second largest operating expense
  – Small improvement in solution quality has significant financial impact

• For decades, researchers have worked on finding better crew scheduling algorithms

• Our approach is to instead improve solution quality by expanding the feasible set of solutions
Airline Schedule Planning

1. Schedule Design
2. Fleet Assignment
   - Aircraft Routing
     - Route individual aircraft honoring maintenance restrictions
   - Crew Scheduling
     - Assign crew (pilots and/or flight attendants) to flight legs
Aircraft Maintenance Routing: Problem Definition

• **Given:**
  
  – Flight Schedule for a single fleet
    • Each flight covered exactly once by fleet
  
  – Number of Aircraft by Equipment Type
    • Can’t assign more aircraft than are available
  
  – Turn Times at each Station
  
  – FAA Maintenance Requirements
    • Maintenance required every 60 hours of flying
    • Airlines maintain aircraft every 40-45 hours of flying with the maximum time between checks restricted to three to four calendar days
Aircraft Maintenance Routing: Objective

• Find:
  – Feasible assignment of individual aircraft to scheduled flights
    • Each flight is covered exactly once
    • Maintenance requirements are satisfied
    • Conservation of flow (balance) of aircraft is achieved
    • The number of aircraft used does not exceed the number available
Example: Maintenance Station in Boston
String Model: Variable Definition

- A string is a sequence of flights beginning and ending at a maintenance station with maintenance following the last flight in the sequence
  - Departure time of the string is the departure time of the first flight in the sequence
  - Arrival time of the string is the arrival time of the last flight in the sequence + maintenance time
String Model: Constraints

- **Maintenance constraints**
  - Satisfied by variable definition

- **Cover constraints**
  - Each flight must be assigned to exactly one string

- **Balance constraints**
  - Needed only at maintenance stations

- **Fleet size constraints**
  - The number of assigned aircraft cannot exceed the number of aircraft in the fleet
Model Solution

• Complex constraints can be handled easily

• Model size
  – Huge number of variables

• Solution approach: branch-and-price
  – Generate string variables only as-needed
Airline Schedule Planning

- Schedule Design
- Fleet Assignment
- Aircraft Routing
- Crew Scheduling

- Route individual aircraft honoring maintenance restrictions
- Assign crew (pilots and/or flight attendants) to flight legs
Integrate?

• Crew scheduling options are limited by maintenance routing decisions made earlier in the airline planning process
• Solving maintenance routing and crew scheduling simultaneously yields a large and challenging problem

➤ Idea is to improve crew scheduling by incorporating relevant maintenance routing decisions
The Maintenance Routing Problem (MR) - find feasible routing of aircraft ensuring adequate aircraft maintenance opportunities and flight coverage.

Crews need enough time between two sequential flights to travel through the terminal -- minimum connect time.

If both flights are covered by the same aircraft, connection time can be reduced.

A short connect is a connection that is crew-feasible only if both flights are assigned to the same aircraft.
Sequential Approach

- **Maintenance Routing Problem**
  - Short Connects flown by the same aircraft

- **Flight Network**
  - Valid pairings

- **Crew Pairing Problem**
Klabjan, Johnson, and Nemhauser

• Solve the crew pairing problem first, including all short connects in the crew pairing network
• Given the crew solution, require all short connects included in it to be part of the maintenance solution, which is solved second
• For “good” instances, this yields the optimal solution to the integrated problem (and many problems are “good”)
• For “bad” instances, this leads to maintenance infeasibility
Cordeau, Stojković, Soumis, Desrosiers

- Directly integrate crew and maintenance routing models
- Basic maintenance routing and crew pairing variables and constraints, plus linking constraints
- Benders decomposition approach using a heuristic branching strategy
- For non-hub-and-spoke networks, positive computational results
Our Approach

• Generate different solutions to the maintenance routing problem
• Allow the crew pairing model to choose the maintenance routing solution with the most useful set of short connects
The Example Again

Flights:
A B C D E F G H
All Possible Short Connects:
A-B A-C E-G

• MR solution (x₁) assigns the same aircraft to short connects A-C and E-G
• MR solution (x₂) assigns the same aircraft to short connect A-B

Potential pairings:
– A-C-D-F (y₁): $1
– A-B-E-F (y₂): $2
– C-D-G-H (y₃): $4
– B-E-G-H (y₄): $6

Crew pairing solutions:
– x₁ => pairings 1, 4: $7
– x₂ => pairings 2, 3: $6
Flights

A → C → D → F
B → E → G → H
Short Connect
Short Connect

A → C → D
E
B

F

G

H
Short Connect
Maintenance Solution 1
Maintenance Solution 2
If MR Solution 1 (A-C, E-G) =>
Optimal: Pairings 1, 4 -- $7
If MR Solution 2 (A-B) =>
Optimal: Pairings 2, 3 -- $6
Approach

• In the sequential approach, given a maintenance routing solution, the crew pairing problem is solved

• We allow the crew scheduler to choose from a collection of maintenance routing solutions
  – Select the one containing the set of short connects that allows the minimum cost crew pairing solution

• Problem: We don’t want to solve one crew pairing problem for each maintenance routing solution

• Solution: Extended Crew Pairing Model (ECP)
The Extended Crew Pairing Model (ECP)

- Simultaneously select a cost minimizing set of crew pairings and a corresponding feasible maintenance routing solution from a given set of maintenance routing solutions
- Add constraints that allow pairings with a short connect to be selected only if the chosen maintenance solution assigns the same aircraft to that short connect
The Example Again

Flights:
A B C D E F G H

Short Connects:
A-B A-C E-G

• MR solution \((x_1)\)
  uses short connects A-C and E-G

• MR solution \((x_2)\)
  uses short connect A-B

• Potential pairings:
  – A-C-D-F  \((y_1): $1\)
  – A-B-E-F  \((y_2): $2\)
  – C-D-G-H  \((y_3): $4\)
  – B-E-G-H  \((y_4): $6\)

• Crew pairing solutions:
  – \(x_1 \Rightarrow \) pairings 1, 4: $7
  – \(x_2 \Rightarrow \) pairings 2, 3: $6
Matrix Representation for the Example

<table>
<thead>
<tr>
<th></th>
<th>x₁</th>
<th>x₂</th>
<th>y₁</th>
<th>y₂</th>
<th>y₃</th>
<th>y₄</th>
<th>rhs</th>
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<tbody>
<tr>
<td>Flights:</td>
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</tr>
</tbody>
</table>

| Short Connects: | 0  | 1  | 0  | −1 | 0  | 0  | ≥ 0 |
|                | 1  | 0  |−1 | 0  | 0  | 0  | ≥ 0 |
|                | 1  | 0  | 0  | 0  | 0  |−1 | ≥ 0 |

| Convexity:     | 1  | 1  | 0  | 0  | 0  | 0  | = 1 |

A  B  C  D  E  F  G  H  A-B  A-C  E-G  Conv.
Notation

- $P_k$ is the set of feasible pairings for fleet family $k$
- $F_k$ is the set of flights assigned to fleet family $k$
- $T_k$ is the set of short connects for the flights assigned to fleet family $k$
- $S_k$ is the set of feasible MR solutions for the flights assigned to fleet family $k$
Notation, cont.

- $\delta_{fp}$ is 1 if flight $f$ is included in pairing $p$, else 0
- $\alpha_{ts}$ is 1 if MR solution $s$ includes short connect $t$, else 0
- $\beta_{tp}$ is 1 if short connect $t$ is contained in pairing $p$, else 0
- $c_p$ is the cost of pairing $p$
Notation, cont.

• $x_s$ is a binary decision variable with value 1 if MR solution $s$ is chosen, else 0

• $y_p$ is a binary decision variable with value 1 if pairing $p$ is chosen, else 0
ECP Formulation

\[
\begin{align*}
\min & \sum_{p \in P^k} c_p y_p \\
\text{st} & \\
\sum_{p \in P^k} \delta_{fp} y_p = 1 & \quad \forall f \in F^k \\
\sum_{s \in S^k} \alpha_{ts} x_s - \sum_{p \in P^k} \beta_{tp} y_p \geq 0 & \quad \forall t \in T^k \\
\sum_{s \in S^k} x_s = 1 \\
x_s, y_p & \in \{0,1\} & \quad \forall s, p
\end{align*}
\]
ECP Enhancements

• By exploiting dominance relationships, can dramatically reduce the number of MR columns considered in finding an optimal ECP solution
  – MR1 containing short connects AB, CD, GH dominates MR2 containing short connect AB
  • Do not need to include MR2 in ECP
• Theoretical bounds and computational observations
  – Example: 61 flights => >> 25,000 MR solutions => 4 non-dominated MR solutions (bounded by 35)
  – Can find these 4 non-dominated MR solutions by solving 4 MR problems
ECP Enhancements, cont.

- **Proof:** Can relax the integrality of MR columns and still achieve integer solutions:
  - Same number of binary variables as original CP

\[
\begin{align*}
\min \sum p c_p y_p \\
st \\
\sum_{p: f \in p} y_p = 1 \quad \forall f \\
\sum_{s \in s} z_s - \sum_{p: t \in P} y_p \geq 0 \quad \forall t \\
\sum_s z_s = 1 \\
\end{align*}
\]

\[
\begin{align*}
\min \sum p c_p y_p \\
st \\
\sum_{p: f \in p} y_p = 1 \quad \forall f \\
\sum_{s \in s} z_s - \sum_{p: t \in P} y_p \geq 0 \quad \forall t \\
\sum_s z_s = 1 \\
\end{align*}
\]

LP relaxation of ECP is tighter than LP relaxation of a basic integrated approach.
Computational Experiment

• **Problem A:**
  
  Lower bound: 31,396.10  
  ECP with 16 MR columns: 31,396.10  
  Optimality gap: 0%

• **Problem B:**
  
  Lower bound: 25,076.60  
  ECP with 20 MR columns: 25,498.60  
  Optimality gap: 1.7%
Airline Crew Scheduling Successes

• Excess crew costs in the planning process has been driven to 0-3%
  – AA was 8-10% 15 yrs ago: now 0-2%
  – Each 1% is worth about $10 million/yr
• 1997 had 9,000 pilots costing $1.2 billion
  – Larger schedules and complex rules
Crew Pre-Month Planning
Ellis Johnson, Georgia Tech

- Crew Pairing Optimization
- Regular/Reserve Bidline Generation
- Bidding and Conflict Resolution
- Supplemental Regular Lines
- Vacation Scheduling
- Initial Training Scheduling
- Recurrent Training Scheduling
- Flight Instructor Scheduling

Month of Operation

Time
Reserve Demand
Ellis Johnson, Georgia Tech

Net Reserve Demand
- Pre-Month Planning
  - (1) Vacation Conflict
  - (2) Initial Training Conflict
  - (3) Transition Conflict
  - (4) Recurrent Training Conflict
- Open Time Trips
  - Could be up to 25% total trips built
- Irregular Operations
  - (1) Weather Disruptions
  - (2) Aircraft Maintenance
  - (3) Sick Leave
- Higher Reserve Availability Desired

Voluntary Flying
Premium Flying
Reserves

Could cover 10% of the open trip flying
Crew: The Rest of The Story
Ellis Johnson, Georgia Tech

• Manpower planning, conflicts, overtime flying, and reserves
• In US airlines as high as 30% of the pilots may be on reserve bid lines
  – Actual flying is about 50% of usual
  – Of that flying, more than half is to cover conflicts and as little as 1/3 is to cover disruptions
Conclusions

• Crew scheduling is critical to airline profitability
  – Making maintenance routing decisions independently increases costs

• A model that fully integrates MR and CP can be inflexible and difficult to solve
  – ECP exploits that only some maintenance routing information is relevant and uses dominance to reduce the size of the problem

• More work to be done… especially post-pairing optimization