Lecture 10

Control of Isolated Traffic Signals

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Lecture 10 Outline

- Isolated saturated intersections
- Definitions: Saturation flow rate, effective green, and lost time
- Notation for an intersection approach variable
- Two assumptions for delay models
- Average delay per vehicle: deterministic term $W_{q,A}$
- Average delay per vehicle: stochastic term $W_{q,B}$
- Webster optimal green time settings: two approaches intersection and numerical example
- Webster cycle time optimization procedure
- Mid-day and evening-peak examples
- Lecture summary
Isolated Saturated Intersections

- An implication of saturation regime: need to efficiently allocate green times ($g_N, g_S$) and ($g_E, g_W$)

Saturation Flow, Effective Green, and Lost Time

- Total lost time $l = l_1 + l_2$ (typically 2 sec)
- Green $(k) + Amber (a) = Effective green time (g) + Total lost time (l) \Rightarrow l = k + a - g$
- Effective green time $(g) \times Saturation flow (s) = Total vehicles discharged during $(k + a)$
Notations for An Intersection Approach

- Webster
  - \( s \): saturation flow rate
  - \( g \): effective green time
  - \( c \): cycle time
  - \( \lambda = \frac{g}{c} \): fraction of effective green in cycle time

<table>
<thead>
<tr>
<th>Webster</th>
<th>Meaning</th>
<th>Queueing Theory</th>
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<tbody>
<tr>
<td>( q )</td>
<td>arrival rate (veh/unit of time)</td>
<td>( \lambda )</td>
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<tr>
<td>( \lambda s )</td>
<td>average flow rate at exit of an approach</td>
<td>( \mu )</td>
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<tr>
<td>( x = \frac{q}{\lambda s} )</td>
<td>degree of saturation</td>
<td>( \rho = \frac{\lambda}{\mu} )</td>
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<tr>
<td>( y = \frac{q}{s} )</td>
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Two Assumptions for Delay Models

- Assumption (A):
  - The interarrival times are constant (view arrivals as evenly spaced at rate \( q \))
  - Service time is constant during effective green and zero in the rest of the cycle
  - Average waiting per vehicle is denoted by \( W_{q,A} \)
- Assumption (B):
  - The interarrival times are exponentially distributed with rate \( q \)
  - Service time is constant with service rate \( \lambda s \)
  - Average waiting per vehicle is denoted by \( W_{q,B} \)
- Webster formula for total waiting time per vehicle:
  \[
  d = W_{q,A} + W_{q,B} - \text{correction factor obtained by simulation}
  \]
Average Delay per Vehicle: Term $\bar{W}_{q,a}$

- Total waiting during $c$ per approach:
  \[
  \frac{1}{2} q(c-g)[(c-g) + \frac{q(c-g)}{s-q}] = \frac{q(c-g)^2}{2} \cdot \frac{s}{s-q} = \frac{q(c-g)^2}{2} \cdot \frac{1}{1-\lambda x}
  \]

- Total arrivals during cycle $c$: $qc$

- $\bar{W}_{q,a} = \left\{ \frac{q(c-g)^2}{2}, \frac{1}{(1-\lambda x)} \right\}, \frac{1}{qc} = \frac{c(1-g/c)^2}{2(1-\lambda x)} = \frac{c(1-\lambda)^2}{2(1-\lambda x)}$

Average Delay per Vehicle: Term $\bar{W}_{q,b}$

- Interarrival times are exponentially distributed with rate $q$, and service times are deterministic with rate $\lambda s$

- Average waiting time for $M/D/1$ queueing system:
  \[
  \frac{1}{2} \cdot \frac{\rho^2}{\lambda(1-\rho)}
  \]

- $\bar{W}_{q,b} = \frac{1}{2} \cdot \frac{(q/\lambda s)^2}{q(1-q/\lambda s)} = \frac{s^2}{2} \cdot \frac{1}{q(1-s)}$

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Webster’s Average Delay Per Vehicle Model

- Average delay per vehicle: \( d = W_{q,a} + \bar{W}_{q,b} - \text{correction term} \)

\[
d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.68 \left( \frac{c}{q^2} \right)^{\frac{1}{3}} x^{\left(\frac{1}{2}+\frac{5x}{3}\right)}
\]

- \( W_{q,a} \) dominates for very small values of \( x \)

- \( W_{q,b} \) dominates for large values of \( x (x \rightarrow 1) \)

- Small value of \( x \) is not an important case from an optimization standpoint

- Optimal green time setting problem: Find \( \lambda_E, \lambda_W, \lambda_N \), and \( \lambda_S \) such that the total delay is minimum

Observed Delay vs. Webster’s Model

Relative delay per vehicle =

Average delay per vehicle / Cycle time

Semi-empirical curve to fit results

Terms \( W_{q,a} + W_{q,b} \)

Term \( W_{q,a} \)

Degree of saturation, \( x \)
“Optimal Settings”: A Two Approaches Intersection

- \( x_1 = \frac{q_1}{\lambda_1 s_1} \), \( x_2 = \frac{q_2}{\lambda_2 s_2} \),

- Note: • (\( q_1, s_1 \)) and (\( q_2, s_2 \)) are given
  • (\( \lambda_1 + \lambda_2 \))c = c
  • \( x_1 \uparrow \), then \( x_2 \downarrow \) and vice versa

- Total delay \( \approx \sum_{i=1}^{2} \frac{x_i^2}{q_i(1-x_i)} \cdot q_i = \frac{1}{2} \sum_{i=1}^{2} \frac{x_i^2}{(1-x_i)} \)

- Minimum total delay: Total delays are about the same on both approaches
  - \( x_1^2 = \frac{x_2^2}{1-x_1} \)
  - \( x_1 = x_2 \)
  - \( \frac{\lambda_2}{\lambda_1} = \frac{g_2/c}{g_1/c} \cdot \frac{g_2}{g_1} \cdot \frac{q_2/s_2}{q_1/s_1} = \frac{y_2}{y_1} \)

Numerical Example 1

- Saturation flow rate \( s = 1800 \text{ veh/hr} \) for all arms (approaches)
  - Lost time \( L = 10 \text{ sec} \)
  - Cycle length \( c = 60 \text{ sec} \)

- \( q_N = q_S = 600 \text{ veh/hr} \); \( q_E = q_W = 300 \text{ veh/hr} \)

- \( y_N = y_S = \frac{600}{1800} = \frac{1}{3} \); \( y_E = y_W = \frac{300}{1800} = \frac{1}{6} \)

- \( y_{N-S} = \frac{1}{3} \); \( y_{E-W} = \frac{1}{6} \)

- \( g_{N-S} = \frac{1}{3} = 2 \)
  - \( g_{E-W} = \frac{1}{6} \)

- \( g_{N-S} + g_{E-W} = 60 - 10 = 50 \text{ sec} \)

- \( 2g_{E-W} + g_{E-W} = 3g_{E-W} = 50 \text{ sec} \Rightarrow g_{E-W} = 50/3 \approx 17 \text{ sec} \); \( g_{N-S} \approx 33 \text{ sec} \)
Cycle Time Optimization

- "Optimal" cycle: \( \ell_o = \frac{1.5L + 5}{1 - y} \)
- \( y = \sum_{i=1}^{n} y_i \), \( n \) = number of phases (typically \( n = 2 \))
- \( L = nl + R \)
  - \( l \) = average time lost per phase (\( l \approx 2 \) sec)
  - \( R \) = all-red time (\( R \approx 6 \) sec)
- Typically \( L \approx 10 \) sec
- Two phases:

Numerical Example 2: Mid-Day

- \( s = 1600 \) veh/hr in each direction (\( N \rightarrow S \); \( S \rightarrow N \); \( E \rightarrow W \); \( W \rightarrow E \))
- 2 phases; all reds = 6 sec/cycle; lost time = 2 sec/phase
- \( q_N = q_E = 600 \) veh/hr; \( q_W = 400 \) veh/hr; \( q_S = 300 \) veh/hr
- \( y_N = y_E = \frac{600}{1600} = \frac{3}{8} \); \( y_W = \frac{400}{1600} = \frac{1}{4} \); \( y_E = \frac{300}{1600} = \frac{3}{16} \)
- \( y_{N.S} = \frac{3}{8} \); \( y_{E.W} = \frac{3}{8} \); \( y = \frac{3}{8} + \frac{2}{8} = \frac{5}{8} \); \( L = 2 \cdot 2 + 6 = 10 \) sec
- \( c_o = \frac{1.5 \times 10 + 5}{1 - \frac{5}{8}} = 53 \) sec; optimal cycle
- \( g_{N,S} = \frac{3}{5} (53 - 10) \approx 26 \) sec; \( g_{E.W} = \frac{2}{5} (53 - 10) = 17 \) sec
- \( x_N = x_E = 0.764 \); \( x_W = 0.779 \); \( x_S = 0.585 \)
- \( \bar{W}_{q,N} = \bar{W}_{q,E} = 18.4 \) sec; \( \bar{W}_{q,W} \approx 28.6 \) sec; \( \bar{W}_{q,E} \approx 20.0 \) sec
- \( \bar{L}_{q,N} = \bar{L}_{q,S} \approx 3.07 \) veh; \( \bar{L}_{q,W} \approx 3.18 \) veh; \( \bar{L}_{q,E} \approx 1.67 \) veh
- Total delay/hr \( \approx 11 \) hours
Numerical Example 2(cont.): Evening-Peak

- $s = 1600$ veh/hr in each direction ($N \rightarrow S; S \rightarrow N; E \rightarrow W; W \rightarrow E$)
- 2 phases; all reds = 6 sec/cycle; lost time = 2 sec/phase
- $q_N = q_S = 800$ veh/hr; $q_W = q_E = 600$ veh/hr
- $y_N = y_S = \frac{800}{1600} = \frac{1}{2}$; $y_W = y_E = \frac{600}{1600} = \frac{3}{8}$
- $y_{N,W} = \frac{1}{2}$; $y_{E,W} = \frac{3}{8}$; $y = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$; $L = 2 \cdot 2 + 6 = 10$ sec
- $c_o = \frac{1.5 \times 10 + 5}{1 - 7/8} = 160$ sec; optimal cycle
- $g_{N,S} \equiv \frac{4}{7}(160 - 10) \approx 86$ sec; $g_{E,W} \equiv \frac{3}{7}(160 - 10) = 64$ sec
- $x_N = x_S = 0.93$; $x_W = x_E = 0.9375$
- $\bar{W}_{q,N} = \bar{W}_{q,S} \approx 62.0$ sec; $\bar{W}_{q,W} = \bar{W}_{q,E} \approx 88.3$ sec
- $\bar{T}_{q,N} = \bar{T}_{q,S} \approx 13.8$ veh; $\bar{T}_{q,W} = \bar{T}_{q,E} \approx 14.7$ veh
- Total delay/hr $\approx 57$ hours

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