Lecture 11 Outline

- Overview of some traffic flow models:
  - Modeling of single link: Car-following models
  - Dynamic macroscopic models of highway traffic
- Dynamic traffic flow management in road networks:
  - Concepts
  - Dynamic traffic assignment
  - Combined dynamic traffic signal control-assignment
- The ACTS Group
**Link Travel Time Models: Car-Following Models**

- **Notation**:

  - Leader
  - Follower

- $x_n(t) - x_{n+1}(t) = \text{spacing (space headway)} = l_{n+1}(t) + \frac{1}{k_{jam}}$

- Speed of vehicle $n$: $\frac{dx_n(t)}{dt} = \dot{x}_n(t)$

- Acceleration (deceleration) of vehicle $n$: $\frac{d^2x_n(t)}{dt^2} = \ddot{x}_n(t)$

- $\dot{x}_n(t) - \dot{x}_{n+1}(t) = \dot{l}_{n+1}(t)$

- Car-following regime: $l_{n+1}(t)$ is below a certain threshold

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**Link Travel Time Models: Car-Following Models**

- **Simple car-following model**:

  $$\ddot{x}_{n+1}(t + T) = a\dot{l}_{n+1}(t) = a(\dot{x}_n(t) - \dot{x}_{n+1}(t))$$

  - $T$: reaction time ($T \approx 1.5$ sec)
  - $a$: sensitivity factor ($a \approx 0.37 s^{-1}$)

- Questions about this simple car-following model:
  - Is it realistic?
  - Does it have a relationship with macroscopic models?
From Microscopic Models To Macroscopic Models

- Simple car-following model: $\ddot{x}_{n+1}(t) = a(\dot{x}_n(t) - \dot{x}_{n+1}(t)) \quad (T = 0)$
- Fundamental diagram: $q = q_{\text{max}} \left(1 - \frac{k}{k_{\text{jam}}} \right)$
- Proof of “equivalency”
  
  \[
  \ddot{x}_{n+1}(y) = a(\dot{x}_n(y) - \dot{x}_{n+1}(y))
  \]

  \[
  \ddot{x}_{n+1}(y)dy = a(\dot{x}_n(y) - \dot{x}_{n+1}(y))dy = a\dot{l}_{n+1}(t)dy
  \]

  \[
  \int_0^{\ddot{x}_{n+1}(y)}dy = \int_0^{a\dot{l}_{n+1}(t)dy}
  \]

  \[
  u_{n+1}(t) - u_{n+1}(0) = a(l_{n+1}(t) - l_{n+1}(0))
  \]

  \[
  u_{n+1}(t) = a l_{n+1}(t) + u_{n+1}(0) - a l_{n+1}(0)
  \]

  If $l_{n+1}(t) = 0$, then $u_{n+1}(t) = 0$ $\Rightarrow$ $u_{n+1}(0) - al_{n+1}(0) = 0$

From Microscopic Model to Macroscopic Model

\[
 u_{n+1}(t) = al_{n+1}(t) = a\left(\frac{1}{k_{n+1}(t)} - \frac{1}{k_{\text{jam}}} \right)
\]

$\Rightarrow u = a\left(\frac{1}{k} - \frac{1}{k_{\text{jam}}} \right)$

$\Rightarrow q = uk = a\left(\frac{1}{k} - \frac{1}{k_{\text{jam}}} \right)k = a \left(1 - \frac{k}{k_{\text{jam}}} \right)$

If $k = 0$, then $q = a$

Since $q = a \geq a \left(1 - \frac{k}{k_{\text{jam}}} \right)$, then $a = q_{\text{max}}$

$\Rightarrow q = q_{\text{max}} \left(1 - \frac{k}{k_{\text{jam}}} \right)$

- Note: if $k \to 0$, then $u \to \infty$. Does this make sense?
Non-linear Car-following Models

\[ \dot{x}_{n+1}(t+T) = a_0 \left( \frac{\dot{x}_n(t) - \dot{x}_{n+1}(t)}{(x_n(t) - x_{n+1}(t))^3} \right)^{1/5} \]

\[ = a_0 \frac{l_{n+1}(t)}{\left( l_{n+1}(t) + \frac{1}{k_{jam}} \right)^{1/5}} \]

If \( T = 0 \), the corresponding fundamental diagram is:

\[ q = u_{max} k \left[ 1 - \left( \frac{k}{k_{jam}} \right)^0.5 \right] \]

Flow Models Derived from Car-Following Models

\[ \dot{x}_{n+1}(t+T) = a_0 \dot{x}_m(t+T) \left( \frac{\dot{x}_n(t) - \dot{x}_{n+1}(t)}{(x_n(t) - x_{n+1}(t))^3} \right)^{1/5} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( m )</th>
<th>Flow vs. Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( q = a_0 \left[ 1 - \left( \frac{k}{k_{jam}} \right) \right] )</td>
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<tr>
<td>1</td>
<td>0</td>
<td>( q = u_{max} k \ln \left( \frac{k_{jam}}{k} \right) )</td>
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<tr>
<td>1.5</td>
<td>0</td>
<td>( q = u_{max} k \left[ 1 - \left( \frac{k}{k_{jam}} \right)^{1.5} \right] )</td>
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<tr>
<td>2</td>
<td>0</td>
<td>( q = u_{max} k \left[ 1 - \left( \frac{k}{k_{jam}} \right) \right] )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( q = u_{max} k \exp \left[ 1 - \left( \frac{k}{k_{jam}} \right) \right] )</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>( q = u_{max} k \exp \left[ \frac{1 - \left( \frac{k}{k_{jam}} \right)}{2} \right] )</td>
</tr>
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Is The Depicted Trajectory Familiar To You, and How To Predict It?

- Two facts:
  - As a vehicle travels along the roadway, its speed might change
  - The speed is lower if surrounding traffic density is higher

- A dynamic traffic model predicts speeds that vary in space and time

- A macroscopic dynamic traffic flow model:
  - Divide the highway into blocks of constant densities
  - Model speeds within and between blocks

### Density-Flow Relationships

- Macroscopic traffic theory shows that there is a relationship between traffic density and flow on a stretch of a highway
- This theory also predicts average vehicle speed
- Theory supported by real-world measurements
- Density-relationships:
  - A concept familiar to you by now
  - An example is depicted on the right
Boundary Propagation: Upstream in Free Flow, Down Stream Congested

1. Two successive density blocks:
   U: upstream block;
   D: the downstream block
2. Depending on flow on each block, there are three cases: Two are shown below

Modeling of Bottlenecks

- Change in density-flow diagram to reflect change in capacity
Downstream in Free Flow and Upstream Congested

A block with maximum flow (critical density) is created in between U and D blocks.

Modeling of Incidents

- Before incident, traffic flow represented by point B.
- An incident acts like a temporary bottleneck if $k_s < k_B < k_D$. Incident reduces flow to a maximum flow given by points d and D.
- Following the clearing of the incident, a new block with density $k_c$ is created.
Traffic Information-Management Systems (TIMS):

- Properties and Methods

- TIMS should be responsive to:
  - “future” demand
  - potential adjustments in travel patterns due to information provision and changes in supply network
  - based on “projected” traffic conditions to:
    - anticipate downstream traffic conditions, and
    - improve credibility

- Predictions methods:
  - Statistical: valid for short intervals only
  - Dynamic traffic assignment methods (models and algorithms)
Dynamic Traffic Flow Methods

- Traffic assignment models:
  - require time-dependent O-D flows
  - incorporate driver behavior, and information provision
  - require link network performance models
  - have high computational requirements

- Three modeling/algorithmic components:
  - Travelers route-choice
  - Prediction of travel times when vehicle paths are known
  - Route-guidance provision

- Two algorithmic components:
  - Path-generation
  - Dynamic traffic assignment

Framework for Dynamic Traffic Assignment Methods

- Dynamic O-D Trip Rates
- Subset of Paths
  - Users’ Route Choice Models
    - dynamic path flow rates
    - path costs
  - Guidance Generation
    - link travel times
    - link volumes
- Path Filter
  - generated paths
  - new paths
  - path costs
- Generation of Optimal Paths
  - generated paths
Types of DTA Models

- Microscopic traffic models (MITSIM):
  - Traffic is represented at the vehicle level
  - Vehicles are moved using car-following and lane changing models

- Mesoscopic traffic models (MesoTS/DynaMIT):
  - Traffic is represented at the vehicle level
  - Speed is obtained using models that relate macroscopic traffic flow variables

- Macroscopic (or flow-based) traffic models:
  - Traffic is represented as continuous variables
  - Speed is obtained using models that relate macroscopic traffic flow variables

- Analytical (flow-based) traffic models: macroscopic models with a good mix of lot of math, computer science and traffic flow understanding

A Small Real-Word Network Model: Amsterdam Beltway Network

- 196 nodes, 310 links, 1134 O-D pairs and 1443 paths
- Morning peak: 2 hours and 20 minutes
- Discretization intervals: 2357 (3.50 sec each)
- Various types of users:
  - Fixed routes
  - Minimum perceived cost routes
  - Minimum experienced cost routes
**Computer Resources Used in Analytical Model**

- Link variables: 25 Mbytes
- Path variables: 34 Mbytes
- Average time for one loading: about 3 minutes
- Saving ratio compared to known analytical methods: 1000
- Results are encouraging for real-time deployment
- Analytical approach: 45 times faster than real time

**Interdependence of Control and Assignment**

- Consequences of the conventional approach:
  - Sub-optimal signal settings;
  - Inconsistent traffic flow predictions.

![Diagram showing the interdependence of Dynamic Traffic Control, Dynamic Traffic Flow, Dynamic Traffic Assignment, Dynamic Signal Setting]
A Case Study (cont.)

- Controls
  - current existing pre-timed control
  - Webster equal-saturation control
  - Smith P₀ Control
  - One-level Cournot control
  - Bi-level Stackelberg control
  - System-optimal Monopoly control

- Route Choices
  - A set of pre-determined paths (4 paths) for each O-D pair
  - Total of 400 paths
  - Demand is model using C-Logit

Results from Back Bay Case Study

<table>
<thead>
<tr>
<th>Controls</th>
<th>Total Travel Time (mins)</th>
<th>Gap from System-Optimum (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing</td>
<td>11784</td>
<td>14.12</td>
</tr>
<tr>
<td>Webster</td>
<td>11781</td>
<td>14.1</td>
</tr>
<tr>
<td>Smith P₀</td>
<td>11566</td>
<td>12.02</td>
</tr>
<tr>
<td>Cournot</td>
<td>10642</td>
<td>3.07</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>10504</td>
<td>1.73</td>
</tr>
<tr>
<td>Monopoly</td>
<td>10325</td>
<td>0</td>
</tr>
</tbody>
</table>
Current Research Interests of ACTS Group

- Algorithms and Computation in Transportation Systems (ACTS) Group:
  - A group formed by Prof. Ismail Chabini and his graduate advisees
  - Has been around for 5 years now

- Field of Interest:
  - Design and computer implementation of models and algorithms for transportation network analysis and operations
  - Modeling and real-time management of traffic flows on road networks
  - Applications to solve increasingly important societal problems such as congestion, air quality, energy, safety, and emergency

- Critical Characteristics of Problems:
  - Real-time operation
  - Dynamics
  - Large networks
  - Intelligence and subsystems integration
  - Complex interactions involving humans

Related Ongoing Research Projects

- NSF CAREER (and NSF ETI): “High Performance Computing and Network Optimization Methods with Applications to Intelligent Transportation Systems” (~ $350k)

- NSF-NYSDOT: “Deployment of ITS technologies” (~ $350k)


- Ford Motor Company: “Emissions Modeling and Control”, and “Parallel Traffic Models and Applications to Safety” (~ $330k)

- CMI (with colleagues from MIT and Cambridge University): “The Sentient Vehicle” (~ $400k)
Some Research Contributions of the ACTS Group:
Routing Problems in Time-Dependent Networks

- Developed the fastest known algorithms for a variety of routing problems in time-dependent networks
- Algorithms are based on the Decreasing-Order-of-Time (DOT) algorithm, described in paper 2.2, which finds all-to-one shortest paths for all times
- Algorithm DOT has been a breakthrough in this area, and was shown to be a degree of magnitude faster than previously known algorithms
- Implementations of algorithm DOT exploiting high performance computing platforms and hierarchical structures of transportation networks resulted in further gains in computational speed
- The developed algorithms are essential building blocks of real-time traffic management methods, and have significantly expanded the boundary of network sizes for which these methods are deployable
- It is now computationally possible to deploy real-time traffic management methods to medium-size networks, which are a degree of magnitude larger

Some Research Contributions of the ACTS Group:
Methods and Tools for Traffic Flow Modeling and Management

- Developed new models, faster algorithms and computer implementations to solve a class of traffic flow modeling and management problems
- They are the fastest known tools in the literature, and run much faster than real-time on medium-size real-life networks
- Developed new traffic management algorithms, which have the potential to significantly improve travel-times through effective real-time dynamic adjustment of traffic signals and provision of route guidance strategies
- The developed models, algorithms and software tools contribute to creating a knowledge and computational base needed to build and deploy real-time traffic management systems to alleviate congestion
- The algorithms and software systems have unique capabilities, and have been sought by industry. Some are the subject of patent applications
- In projects sponsored by CMI and Ford-MIT initiatives, the developed methods are being used to design new solutions to problems related to air pollution, energy consumption, and safety
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