Lecture 2

Modeling Air Traffic Flows

Prof. Ismail Chabini
Prof. Amedeo R. Odoni

Lecture 2 Outline

- Background
- Factors that ‘Determine’ a Runway Capacity
- Minimum Time Separation between Two Arriving Aircrafts
- Arrival Capacity Model for One Runway
- Example
- Practical Issues and Model Analysis
- Other Models
- Summary
Background

- Two main types of air transportation infrastructure:
  - Airports
  - Air Traffic Control (ATC)
- Airport services and facilities:
  - Airside or Airfield (runways, taxiways, hangers)
  - Landside (terminals, parking areas, access roads)
- Models deal with issues related to:
  - Passenger terminals
  - Runway and taxiway systems
  - ATC
- Types of models:
  - Analytical (i.e., the model of this lecture)
  - Simulation (will be covered in a later lecture)
Arrival runway capacity is (partially) determined by:

- Rules of Traffic Flow:
  - Minimum separation between two successive aircraft
  - Only one aircraft can be on the runway at any time

- Aircraft Population:
  - Types: Heavy, Large/Medium, Small
  - Mix: Percentage of each type

Arrival Capacity Model of a Single Runway

- Idealized representation:

- Notation:
  - $n$: length of final approach path
  - $i$ ($j$): type of leading (trailing) aircraft
  - $v_i$: ground speed of type $i$ aircraft
  - $o_i$: runway occupancy time of type $i$ aircraft
  - $s_{ij}$: minimum separation between two airborne aircraft
  - $T_{ij}$: minimum acceptable time interval between successive arrivals at the runway of type $i$ and type $j$ aircrafts (unknown)
Minimum Time Separation of Two Aircrafts

Minimum time separation is a consequence of:
- Minimum space separation must not be violated
- Only a single aircraft can be on the runway at a given time instance

Question: What is the expression of minimum time separation $T_{ij}$?

$T_{ij} > o_i$

Answer:

$$T_{ij} = \begin{cases} \max \left( \frac{n+s_{ij}}{v_j}, \frac{n}{v_j}, o_i \right) \quad \text{for } v_i > v_j \quad \text{('opening case')} \\ \max \left( \frac{s_{ij}}{v_j}, o_i \right) \quad \text{for } v_i \leq v_j \quad \text{('closing case')} \end{cases}$$

Arrival Capacity of a Single Runway

- ‘opening case’: $v_i > v_j$
- ‘closing case’: $v_i \leq v_j$
Minimum Acceptable Interarrival Time

\[ T_y = \begin{cases} 
\max \left( \frac{n + s_j}{v_j}, \frac{n}{v_i}, o_i \right) & \text{for } v_i > v_j \quad (\text{'opening case'}) \\
\max \left( \frac{s_j}{v_i}, o_i \right) & \text{for } v_i \leq v_j \quad (\text{'closing case'})
\end{cases} \]

- **\( K \): number of aircraft types**
- Number of ‘type \( i \) aircraft followed by type \( j \) aircraft’ pairs = \( K^2 \)
- **\( p_{ij} \): probability of ‘type \( i \) aircraft followed by type \( j \) aircraft’ pair**
- Minimum acceptable interarrival time: \( E[T_y] = \sum_{i,j} \sum_{y} p_{ij} \times T_{ij} \)

### Example

#### Parameters (given)
- \( i \) (a/c type) | \( p_i \) (prob.) | \( v_i \) (knots) | \( o_i \) (sec)
  - 1(H) | 0.2 | 150 | 70
  - 2(L) | 0.35 | 130 | 60
  - 3(M) | 0.35 | 110 | 55
  - 4(S) | 0.1 | 90 | 50

#### \([s_{ij}]\) matrix (given)

\[
T = \begin{bmatrix}
96 & 157 & 207 & 320 \\
60 & 69 & 107 & 222 \\
60 & 69 & 82 & 196 \\
60 & 69 & 82 & 100
\end{bmatrix}
\]

#### \([p_{ij}]\) matrix:
- \( p_{ij} = p_i p_j \)

#### \([s_{ij}]\) matrix:

\[
S = \begin{bmatrix}
4 & 5 & 5 & 6 \\
2.5 & 2.5 & 2.5 & 4 \\
2.5 & 2.5 & 2.5 & 4 \\
2.5 & 2.5 & 2.5 & 2.5
\end{bmatrix}
\]

- **\( K \) symmetric matrix**
- **\( s_{ij} \): trailing aircraft**
- **\( p_{ij} \): leading aircraft**

\[
T = \begin{bmatrix}
1(H) & 2(L) & 3(M) & 4(S) \\
1(H) & 2(L) & 3(M) & 4(S)
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
0.04 & 0.07 & 0.07 & 0.02 \\
0.07 & 0.1225 & 0.1225 & 0.035 \\
0.07 & 0.1225 & 0.1225 & 0.035 \\
0.02 & 0.035 & 0.035 & 0.01
\end{bmatrix}
\]

\[
E[T_y] = \sum_{i,j} \sum_{y} p_{ij} \times T_{ij} = 106.3 \text{ sec}
\]
**Arrival Runway Capacity: Example**

- **Maximum flow rate** (frequency of arrivals): \( \frac{1}{E[T_{ij}]} = 33.9 \) arrivals/hour
  \( \Rightarrow \) It is called maximum theoretical capacity.

- In practice, minimum separation is: \( T'_{ij} = T_{ij} + b \)

- In practice, \( \frac{1}{E[T'_{ij}]} \) is called the ‘saturation capacity’ or ‘maximum throughput’.

- Typically, \( b = 10 \text{ sec} \) \( \Rightarrow \frac{1}{E[T'_{ij}]} = \frac{1}{116.3} = 30.9 \) aircrafts/hour

---

**Comments on Analytical Model**

- The analytical model is simple, but insightful!

- It assesses impacts on capacity of changes in ATC rules and ATC operational conditions and methods

- Example of possible changes to increase capacity:
  - Reduce separation
  - Increase final approach speed
  - Length of approach path (increase? or decrease?)
  - Change mix by forbidding or pricing out small planes
  - Sequencing of aircrafts that are waiting to land (operational problem)
### Analysis of the Analytical Model

- Limitations of the model:
  - $v_i$ and $o_i$ are random in practice, but assumed constant in the model
  - Distance between aircrafts is random in practice, but not in the model
  - The model assumes an isolated runway dedicated for landing only

- Other complications in capacity modeling:
  - Airports contain and operate under multiple runways (PS1)
  - A given set of runways is operated using multiple configurations
  - The use of a configuration depends on:
    - Level of demand
    - Weather conditions
    - Wind speed and direction
    - Traffic mix
  - A runway can be used for both arrivals and departures (PS1)

### Other Models in Air Transportation

- Airport passenger terminal design
- Simulation models of the airfield
- Analysis of air delay using analytical or simulation models (see later lectures)
- Estimation of the frequency of ‘conflicts’ resolution:
  - Estimation of number of overtaking conflicts involving aircrafts flying on the same airway
  - Prediction of the number of conflicts at the intersection of two airways
  - Estimates given by these models are principal indicators of the workload of an ATC system
Lecture 2 Summary

- Background
- Factors that ‘Determine’ a Runway Capacity
- Minimum Time Separation between Two Arriving Aircrafts
- Arrival Capacity Model for One Runway
- Example
- Practical Issues and Model Analysis
- Other Models