Outline

1. Wait time models
2. Service variation along route
3. Running time models
Simple deterministic model:

\[ E(w) = E(h)/2 \]

where

\[ E(w) = \text{expected waiting time} \]
\[ E(h) = \text{expected headway} \]

Model assumptions:

- passenger arrival times are independent of vehicle departure times
- vehicles depart deterministically at equal intervals
- every passenger can board the first vehicle to arrive
Passenger Arrival Process

- Individual, group, and bulk passenger arrivals
- Passengers can be classified in terms of arrival process:
  - random arrivals
  - time arrival to minimize $E(w)$
  - arrive with the vehicle, i.e. have $w = 0$
For long headway service have “schedule delay” as well as wait time.

\[ E(w) = \frac{E(h)}{2} \]
Vehicle Departure Process

Vehicle departures typically not regular and deterministic

Wait Time Model refinement:

If:

\[ n(h) = \# \text{ of passengers arriving in a headway } h \]

\[ \bar{w}(h) = \text{mean waiting time for passengers arriving in headway } h \]

\[ g(h) = \text{probability density function of headway} \]

Then:

\[ E(w) = \text{Expected Total Passenger Waiting Time per vehicle departure} \]

\[ \frac{\int_{0}^{\infty} n(h)\bar{w}(h)g(h)dh}{\int_{0}^{\infty} n(h)g(h)dh} \]
Vehicle Departure Process

Now if:

\[ n(h) = \lambda \cdot h \text{ where } \lambda \text{ is passenger arrival rate} \]

\[ \bar{w}(h) = \frac{h}{2} \]

Then:

\[ E(w) = \frac{E(h^2)}{2E(h)} = \frac{E(h)}{2} \left[ 1 + \frac{\text{var}(h)}{(E(h))^2} \right] = \frac{E(h)}{2} \left[ 1 + (\text{cov}(h))^2 \right] \]
Vehicle Departure Process Examples

A. If $\text{var}(h) = 0$:

$$E(w) = \frac{E(h)}{2}$$

B. If vehicle departures are as in a Poisson process:

$$\text{var}(h) = (E(h))^2 \text{ and } E(w) = E(h)$$

C. The headway sequence is 5, 15, 5, 15, ... then:

$$E(h) = 10$$

$$E(w) = 2.5 \times 0.25 + 7.5 \times 0.75 = 6.25 \text{ mins}$$
Passenger Loads Approach Vehicle Capacity

- Not all passengers can board the first vehicle to depart:

- General queuing relationship
Service Variation Along Route

- Service may vary along route even without capacity becoming binding:
- the headway distribution can vary along the route, affecting $E(w)$
- at the limit vehicles can be paired, or bunched
- this can also result in passenger load variation between vehicles
Service Variation Along Route (cont’d)

Distance along route

Stop 3

Stop 2

Departure point

Time

dep. 1  dep. 2  dep. 3

more pass.

pairing

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Spring 2010, Lecture 14
Service Variation Along Route (cont’d)

$\text{pdf}(h)$

- at start of route
- at end of route
- midpoint of route

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Factors Affecting Headway Deterioration

• Length of route
• Marginal dwell time per passenger
• Stopping probability
• Scheduled headway
• Driver behavior

Simple model:

\[ e_i = (e_{i-1} + t_i) \left(1 + p_{i-1} \cdot b \right) \]

where

- \( e_i \) = headway deviation (actual-scheduled) at stop \( i \)
- \( t_i \) = travel time deviation (actual-scheduled) from stop \( i-1 \) to \( i \)
- \( p_i \) = passenger arrival rate at stop \( i \)
- \( b \) = boarding time per passenger
Mathematical Model for Headway Variance*

\[
\text{var}(h_i) = \text{var}(h_{i-1}) + \text{var}(\Delta t_{i-1}) + 2p_{i-1}(1 - p_{i-1})(c \cdot \bar{q}_{i-1} + \ell)^2
\]

\[
+ 2c^2 \text{var}(q_{i-1}) \left[1 - \rho_q + p_{i-1}\rho_q\right](1 - p_{i+1})
\]

\[
+ c(1 - p_{i-1})^2 \cdot \text{cov}(\Delta q_{i-1}, h_{i-1})
\]

where:

- \text{var}(h_i) = \text{headway variance at stop } i
- \text{var}(\Delta t_i) = \text{variance of the difference in running time between successive buses between stops } i - 1 \text{ and } i
- p_i = \text{probability bus will skip stop } i
- c = \text{marginal dwell time per passenger served at a stop}
- \bar{q}_i = \text{mean number of passengers per bus served at stop } i
- \ell = \text{the constant term of the dwell time function}
- \text{var}(q_i) = \text{variance of the number of passengers served per bus at stop } i
- \rho_q = \text{correlation coefficient between the passengers served by successive buses at a stop}
- \text{cov}(\Delta q_i, h_i) = \text{covariance of the difference in number of passengers served by successive buses and the headway at stop } i

Vehicle Running Time Models

Different levels of detail:

A. Very detailed, microscopic simulation:
   • represents vehicle motion and interaction with other vehicles, e.g. buses operating in mixed traffic, or train interaction through control system

B. Macroscopic:
   • identify factors which might affect running times
   • collect data and estimate model
Running Time includes dwell time, movement time, and delay time:

- dwell time is generally a function of number of passengers boarding and alighting as well as technology characteristics
- movement time and delay depend on other traffic and control system attributes

**Typical bus running time breakdown in mixed traffic:**
- 50-75% movement time
- 10-25% stop dwell time
- 10-25% traffic delays including traffic signals