MACRO DESIGN MODELS FOR A SINGLE ROUTE

Outline
1. Introduction to analysis approach
2. Bus frequency model
3. Bus size model
4. Stop/station spacing model
Introduction to Analysis Approach

- Basic approach is to establish an aggregate total cost function including:
  - operator cost as \( f(\text{design parameters}) \)
  - user cost as \( g(\text{design parameters}) \)
- Minimize total cost function to determine optimal design parameter (s.t. constraints)

Variants include:
- Maximize service quality s.t. budget constraint
- Maximize consumer surplus s.t. budget constraint
Bus Frequency Model: the Square Root Model

Problem: define bus service frequency on a route as a function of ridership

Total Cost = operator cost + user cost

\[ Z = c \cdot \frac{t}{h} + b \cdot r \cdot \frac{h}{2} \]

where

\[ Z = \text{total (operator + user) cost per unit time} \]
\[ c = \text{operating cost per unit time} \]
\[ t = \text{round trip time} \]
\[ h = \text{headway – the decision variable to be determined} \]
\[ b = \text{value of unit passenger waiting time} \]
\[ r = \text{ridership per unit time} \]

Minimizing \( Z \) w.r.t. \( h \) yields:

\[ h = \sqrt{\frac{2ct}{br}} \text{ or } \sqrt{\frac{2}{b} \left( \frac{c}{b} \right) \left( \frac{t}{r} \right)} \]
This is the Square Rule with the following implications:

- high frequency is appropriate where \((\text{cost of wait time}/\text{cost of operations time})\) is high
- frequency is proportional to the square root of ridership per unit time for routes of similar length
• Load factor is proportional to the square root of the product of ridership and route length.
Critical Assumptions:
• bus capacity is never binding
• only frequency benefits are wait time savings
• ridership $\neq f$ (frequency)
• simple wait time model
• budget constraint is not binding

Possible Remedies:
• introduce bus capacity constraint
• modify objective function
• introduce $r=f(h)$ and re-define objective function
• modify objective function
• introduce budget constraint
If: \[ c = \$90/\text{bus hour} \]
\[ b = \$10/\text{passenger hour} \]
\[ t = 90 \text{ mins} \]
\[ r = 1,000 \text{ passengers/hour} \]

Then: \[ h_{OPT} = 11 \text{ mins} \]
Problem: define optimal bus size on a route

Assumptions:
- Desired load factor is constant
- Labor cost/bus hour is independent of bus size
- Non-labor costs are proportional to bus size
- Bus dwell time costs per passenger are independent of bus size

Using same notation as before plus:
- \( w \) = labor cost per bus hour
- \( p \) = passenger flow past peak load point
- \( k \) = desired bus load - the decision variable to be determined

\[
Z = w \cdot \frac{t}{h} + b \cdot r \cdot \frac{h}{2}
\]

Now \( h = \frac{k}{p} \) by assumption above

\[
Z = \frac{wtp}{k} + \frac{brk}{2p}
\]

Minimizing \( Z \) w.r.t. \( k \) gives:

\[
k_{opt} = \sqrt{\frac{2p^2wt}{rb}}
\]
Result is another square root model, implying that optimal bus size increases with:

- round trip time
- ratio of labor cost to passenger wait time cost
- peak passenger flow
- concentration of passenger flows

Previous example extended with:

\[ p = 500 \text{ pass/hour} \]
\[ w = 40\text{/bus hour} \]

all other parameters as before:

Then:

\[ h_{\text{OPT}} = 55 \]
Stop/Station Spacing Model

Problem: determine optimal stop or station spacing

Trade-off is between walk access time (which increases with station spacing), and in-vehicle time (which decreases as station spacing increases) for the user, and operating cost (which decreases as station spacing increases)

Define

\[ Z = \text{total cost per unit distance along route and per headway} \]

and

\[ T_{st} = \text{time lost by vehicle making a stop} \]

\[ c = \text{vehicle operating cost per unit time} \]

\[ s = \text{station/stop spacing - the decision variable to be determined} \]

\[ N = \text{number of passengers on board vehicle} \]

\[ v = \text{value of passenger in-vehicle time} \]

\[ D = \text{demand density in passenger per unit route length per headway} \]

\[ v_{acc} = \text{value of passenger access time} \]

\[ w = \text{walk speed} \]

\[ c_s = \text{station/stop cost per headway} \]
Stop/Station Spacing Model (cont’d)

\[ Z = \frac{T_{st}}{s} (c + N \cdot v) + \frac{c_s}{s} + \frac{s}{4} D \cdot \frac{v_{acc}}{w} \]

Minimizing \( Z \) w.r.t. \( s \) gives:

\[ s_{opt} = \left[ \frac{4w}{Dv_{acc}} \left[ c_s + T_{st} (c_v + Nv) \right] \right]^{1/2} \]

Yet another square root relationship, implying that station/stop spacing increases with:

- walk speed
- station/stop cost
- time lost per stop
- vehicle operating cost
- number of passengers on board vehicle
- value of in-vehicle time

and decreases with:

- demand density
- value of access time
Bus Stop Spacing

**U.S. Practice**
- 200 m between stops (8 per mile)
- shelters are rare
- little or no schedule information

**European Practice**
- 320 m between stops (5 per mile)
- named & sheltered
- up to date schedule information
- scheduled time for every stop
Stop Spacing Tradeoffs

- Walking time
- Riding time
- Operating cost
- Ride quality

![Graph showing the relationship between stop spacing (m) and operator + user cost, with curves for extra walk time, extra operating cost, extra riding time, and total extra cost.](image)
Walk Access: Block-Level Modeling

Figure by MIT OpenCourseWare.
Results: MBTA Route 39*

AM Peak Inbound results

- Avg walking time up 40 s
- Avg riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses

Bus Stop Locations and Policies

- **Far-side** (vs. Near-side)
  - less queue interference
  - easier pull-in
  - fewer ped conflicts
  - snowbank problem demands priority in maintenance

- **Curb extensions** benefit transit, peds, and traffic (0.9 min/mi speed increase)

- **Pull-out priority** (it’s the law in some states)

- **Reducing dwell time** (vehicle design, fare collection, fare policy)