Outline

1. Timetable Development
2. Fleet Size
3. Vehicle Scheduling
Can translate frequency into timetable by specifying headways as:

- **equal** -- appropriate if demand is uniformly distributed across period
- **balanced load** -- appropriate if there is substantial variation in demand over period
- **clockface or not** -- do headways repeat every hour
If we have \( N \) departures in peak period:

- **equal headway solution:**
  
  \[
  H = \frac{\text{Peak Period}}{N}
  \]

- **balanced load solution:**

  \[
  \text{Pass Load / Departure} = \frac{\text{Total Passenger Flow}}{N}
  \]
Salzborn’s Fleet Size Theorem:

Given:

\[ l(k,t,s) = \text{# of departures from terminal } k \text{ by time } t \text{ following schedule } s \]
\[ a(k,t,s) = \text{# of arrivals at terminal } k \text{ by time } t \text{ following schedule } s \]

and:

\[ d(k,t,s) = l(k,t,s) - a(k,t,s), \text{ deficit function at terminal } k \text{ at time } t \text{ following schedule } s \]
Salzborn’s Fleet Size Theorem:

Then:

\[ N(s), \text{ the minimum size fleet to serve schedule } s, \text{ is given by:} \]

\[ N(s) = \sum_{k \in T} \max_t (d(k,t,s)) \]

Also, \( N(s) \geq \text{Max # of trips in simultaneous operation.} \)
The deficit function, or minimum required fleet size, may be reduced by:

- shifting departure and/or arrival times
- adding deadhead trips between terminals
Vehicle Scheduling Problem

Input:

-- A set of vehicle revenue trips to be operated, each characterized by:
   -- starting point and time
   -- ending point and time

-- Possible layover arcs between the end of a trip and the start of a (later) trip at the same location

-- Possible deadhead arcs connecting:
   -- depot(s) to trip starting points
   -- trip ending points to depot(s)
   -- trip ending points to trips starting at a different point
Observations:

-- there are many feasible but unattractive deadhead and layover arcs, generate only plausible non-revenue arcs

-- layover time affects service reliability, set minimum layover (recovery) time
Objective:

-- Define vehicle blocks (sequences of revenue and non-revenue activities for each vehicle) covering all trips so as to:

  -- minimize fleet size (i.e. minimize #crews)
  -- minimize non-revenue time (i.e. minimize extra crew time)

Observation:

-- These are proxies for cost, but a large portion of cost will depend on crew duties which are unknown at this stage of solution.
Vehicle Scheduling Problem (continued)

Constraints:
-- Minimum vehicle block length
-- Maximum vehicle block length

Variations:
-- each vehicle restricted to a single line vs. interlining permitted
-- single depot vs multi-depot
-- vehicle fleet size constrained at depot level
-- routes (trips) assigned to specific depot
-- multiple vehicle types
Example: Single Route AB

A

B (Central City)

Results of earlier planning and scheduling analysis:

<table>
<thead>
<tr>
<th></th>
<th>AM Peak Period (6-9 AM)</th>
<th>Base Period (after 9 AM)</th>
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<tbody>
<tr>
<td>Headways</td>
<td>20 min</td>
<td>30 min</td>
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<tr>
<td>Scheduled trip time</td>
<td>40 min</td>
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<td>(A⇒B or B⇒A)</td>
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<td>Minimum layover time</td>
<td>10 min</td>
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Dominant direction of travel in AM is A⇒B
## Timetable and Vehicle Block Development

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# Timetable and Vehicle Block Development

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### Timetable and Vehicle Block Development

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x = from depot

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Example: Vehicle Blocks

**Block 1:** Depot - A (6:00) - B (6:50) - A (7:40) - B (8:30) - A (9:30) - B (10:15) - A (11:00) - B (11:45) - ...

**Block 2:** Depot - A (6:20) - B (7:10) - A (8:00) - B (8:50) - Depot

**Block 3:** Depot - A (6:40) - B (7:30) - A (8:20) - B (9:15) - A (10:00) - B (10:45) - ...

**Block 4:** Depot - A (7:00) - B (7:50) - A (8:40) - Depot

**Block 5:** Depot - A (7:20) - B (8:10) - A (9:00) - B (9:45) - A (10:30) - B (11:15) - ...
Heuristic approaches:

1. Define compatible trips at same terminal $k$ such that trips $i$ and $j$ are compatible iff:

\[
\begin{align*}
    t_{sj} - t_{ei} &> M_k \\
    t_{sj} - t_{ei} &< 2 D_k
\end{align*}
\]

where

- $t_{sj}$ = starting time for trip $j$
- $t_{ei}$ = ending time for trip $i$
- $M_k$ = minimum recovery/layover time at terminal $k$
- $D_k$ = deadhead time from terminal $k$ to depot
2. Apply Restricted First-in-First-out rules at each terminal

a) Start with (next) earliest arrival at terminal; if none, go to step (d)

b) Link to earliest compatible trip at terminal; if none, return vehicle to depot and return to step (a)

c) Check vehicle block length against constraint: if constraining, return vehicle to depot and return to step (a); otherwise return to step (b) with new trip arrival time

d) Serve all remaining unlinked departures from depot
Time-Space Network Representation

Route 1

A_1
B_1

Depot

— revenue arc

Time of Day

Route N

A_N
B_N

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Time-Space Network Representation

- Revenue arc
- Layover arc

Route 1

Route N

Depot

Time of Day

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Time-Space Network Representation

Route 1

A<sub>1</sub> --> B<sub>1</sub> --> Depot

Route N

A<sub>N</sub> --> B<sub>N</sub> --> Depot

--- revenue arc
----- layover arc
------ deadhead arc

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