Performance Models

Outline
1. Wait time models
2. Service variation along route
3. Running time models
4. Dwell time models

Passenger Arrival Process
- Individual, group, and bulk passenger arrivals
- Passengers can be classified in terms of arrival process
  - random arrivals
  - time arrival to minimize $E[W]$
  - arrive with the vehicle, i.e. have $W = 0$

Vehicle Departure Process
Vehicle departures typically not regular and deterministic
Wait Time Model refinement:

$w(h) = \text{mean waiting time for passengers arriving in headway } h$

$g(h) = \text{probability density function of headway}$

\[
E[W] = \frac{\text{Expected Total Passenger Waiting Time per Vehicle Departure}}{\text{Expected Passengers per Vehicle Departure}} = \frac{\int_{0}^{\infty} n(h) \cdot w(h) \cdot g(h) \cdot dh}{\int_{0}^{\infty} n(h) \cdot g(h) \cdot dh}
\]
Vehicle Departure Process

\[ n(h) = \lambda h \quad \text{where } \lambda \text{ is the passenger arrival rate} \]

\[ \bar{w}(h) = \frac{h}{2} \]

\[ E[W] = \frac{E[h^2]}{2E[h]} = \frac{E[h]}{2} \left[ 1 + \frac{\text{Var}[h]}{(E[h])^2} \right] = \frac{E[h]}{2} \left[ 1 + c_h^2 \right] \]

where \( c_h \) is coefficient of variation of headway

Passenger Loads Approach Vehicle Capacity

- Not all passengers can board the first vehicle to depart:

Service Variation Along Route

- Service may vary along route even without capacity becoming binding:
  - the headway distribution can vary along the route, affecting \( E[W] \)
  - at the limit vehicles can be paired, or bunched
  - this can also result in passenger load variation between vehicles

Vehicle Departure Process Examples

If \( \text{Var}[h] = 0 \)

\[ E[W] = \frac{E[h]}{2} \]

If vehicle departures are as in a Poisson process

\[ \text{Var}[h] = (E[h])^2 \quad E[W] = E[h] \]

If the headway sequence is 5, 15, 5, 15, ...

\[ E[h] = 10 \]

\[ E[W] = 2.5 \cdot 0.25 + 7.5 \cdot 0.75 = 6.25 \text{ minutes} \]
Factors Affecting Headway Deterioration

- Length of route
- Marginal dwell time per passenger
- Stopping probability
- Scheduled headway
- Driver behavior

Simple model \[ e_i = (e_{i-1} + t_i)(1 + p_{i-1}b) \]

where
- \( e_i \) = headway deviation (actual - scheduled) at stop \( i \)
- \( t_i \) = travel time deviation (actual - scheduled) from stop \( i-1 \) to \( i \)
- \( p_i \) = passenger arrival rate at stop \( i \)
- \( b \) = boarding time per passenger

Mathematical Model for Headway Variance

\[
\begin{align*}
\text{var}(h_i) &= \text{var}(h_{i-1}) + \text{var}(\Delta t_{i-1}) + 2p_i(1-p_i)(c+\ell)^2 + 2c^2\text{var}(\Delta q_i)(1-p_i+p_i(1-p_i)) + c(1-p_i)^2 \cdot \text{cov}(\Delta q_{i-1}, h_i) \\
\text{var}(\Delta t) &= \text{variance of the difference in running time between successive buses between stops } i-1 \text{ and } i \\
p_i &= \text{probability bus will skip stop } i \\
c &= \text{marginal dwell time per passenger served at a stop} \\
\bar{q}_i &= \text{mean number of passengers per bus served at stop } i \\
\ell &= \text{the constant term of the dwell time function} \\
\text{var}(\Delta q_i) &= \text{variance of the number of passengers served per bus at stop } i \\
p \rho &= \text{correlation coefficient between the passengers served by successive buses at a stop} \\
\text{cov}(\Delta q_{i-1}, h_i) &= \text{covariance of the difference in number of passengers served by successive buses and the headway at stop } i
\end{align*}
\]

Vehicle Running Time Models

Different levels of detail

- Very detailed, microscopic simulation
  - represents vehicle motion and interaction with other vehicles
  - buses operating in mixed traffic
  - train interaction through control system
- Macroscopic
  - identify factors which might affect running times
  - collect data and estimate model

Vehicle Running Time Models

- Running time includes dwell time, movement time, and delay time
  - dwell time is generally a function of number of passengers boarding and alighting as well as technology characteristics
  - movement time and delay depend on other traffic and control system attributes
- Typical bus running time breakdown in mixed traffic
  - 50-75% movement time
  - 10-25% stop dwell time
  - 10-25% traffic delays including traffic signals

Dwell Time Models

- Dwell Time Theory
- Bus Dwell Time Model
- Light Rail Dwell Time Model
- Heavy Rail Dwell Time Model
  - Puong, A., "Dwell Time Model and Analysis for the MBTA Red Line." Internal memo, MIT, March 2000.

Dwell Time Theory

- Vehicle dwell time affects
  - system performance
  - service quality
- A critical element in vehicle bunching resulting in
  - high headway variability
  - high passenger waiting times
  - uneven passenger loads
- Dwell time impact on performance depends on:
  - stop/station spacing
  - mean dwell as proportion of trip time
  - mean headway
  - operations control procedures
- Examples
  - Commuter rail → little impact of dwell time on performance
  - Long, high-frequency bus route → major impact
**Dwell Time Theory**

- Dwell time depends on many factors
  - human
  - modal
  - operating policies & practices
  - weather
- For a given system we have the following possible models
  - Single door, no congestion and interference
    \[ DOT = a + b(DONS) + c(DOFFS) \]
  - Single door with congestion and interference
    \[ DOT = a + b(DONS) + c(DOFFS) + d(DONS+DOFFS)(STD) \]
  - Single car with \(m\) doors
    \[ DT = \max(DOT_1, \ldots, DOT_m) \]
  - Single car with \(m\) doors, with balanced flows
    \[ DT = a + b/m(CONS) + c/m(COFFS) + d/m(CONS+COFFS)(STD) \]
  - \(n\)-car train
    \[ DT = \max(DT_1, \ldots, DT_n) \]
  - \(n\)-car train, with balanced flows
    \[ DT = a + b/nm(TONS) + c/nm(TOFFS) + d/nm(TONS+TOFFS)(STD) \]

**Bus Dwell Time: Prior Work**

- Manually collected data
  - Limited data on infrequent events
  - Crowding
  - Do not include latest fare media
- Automatically collected data
  - Does not include fare media information
  - Poor fit of model
- Transit Capacity and Quality of Service Manual
  - Assumes a half-second penalty per passenger for crowding

**Objective**

- Develop a dwell time model using automatically collected data
- Dwell time factors
  - Boarding and alighting passengers
  - Onboard passengers
  - Fare media type
  - Alighting door selection
  - Bus type
- Minimize the unexplained variation in dwell time
- Evaluate impact on dwell time of:
  - fare media type
  - bus design
  - enforcement of rear-only alightings

**Data Set**

- Automatically collected data from Chicago Transit Authority bus network
- Non-Timepoint, Far-Side, Known Stops
- Functioning APC counters on all doors
  - Verified by non-zero counts across day
  - Minimum per-passenger dwell time of .5 seconds
- Link-in AFC transactions
  - Fare transactions that take place within the dwell time
- Data from entire month of November 2006
  - 173,750 Records
  - 2,977 Operators
  - 85 Routes
  - 927 Stops
Model Formulation

- Predict dominant door activity
- Segment data and compare by:
  - Bus type
  - Crowding (passengers > number of seats)
- Combine the data and test for significant differences in the estimators

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Dwell Time Estimates – Rear Door

<table>
<thead>
<tr>
<th>Variable</th>
<th>DUMMY</th>
<th>est</th>
<th>t-stat</th>
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Adjusted R²: 0.37

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Dwell Time Estimates – Front Door

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<td>FOFF3UP</td>
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<td>CARDS</td>
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</table>

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Bus Dwell Time Model: Key Findings

- Smart media loses benefit in crowded conditions
  - Drops from 2 second advantage in non-crowded conditions
- Crowding impact increases exponentially
- Bus attributes impact dwell time
  - Location of magnetic stripe reader (half second difference)
  - Double-wide doors
- Front door alightings may affect dwell time, while rear door alightings will happen in parallel

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MBTA Green Line Analysis

- Branching network of 28 miles (45 km) and 70 stations
- 52-seat ALRVs operate in 1-, 2-, and 3-car trains
  - high floor, low platform configuration
  - 3 doors per car on each side
  - single side boarding/alighting
- Trunk service in central subway:
  - 10 or 14 stations on round-trip
  - 1- to 2-minute headways
  - peak flows ≈ 10,000 passengers/hour

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Models with Crowding Term

- One-car trains
  - $DT = 12.50 + 0.55^{*}TONS + 0.23^{*}TOFFS + 0.0078^{*}SUMASLS$
    - (8.94) (3.76) (2.03) (6.70)
    - $R^2 = 0.62$
  - $SUMASLS = TOFFS^{*}AS + TONS^{*}LS$
- Two-car trains
  - $DT = 13.93 + 0.27^{*}TONS + 0.36^{*}TOFFS + 0.0008^{*}SUMASLS$
    - (7.43) (2.92) (3.79) (2.03)
    - $R^2 = 0.70$

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Predicted Dwell Times

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<thead>
<tr>
<th>ONS</th>
<th>LPL</th>
<th>1-Car DT</th>
<th>2-Car DT</th>
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<td>0</td>
<td>any #</td>
<td>12.5</td>
<td>13.9</td>
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<tr>
<td>10</td>
<td>&lt; 53</td>
<td>20.3</td>
<td>20.2</td>
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<td>10</td>
<td>150</td>
<td>35.6</td>
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<tr>
<td>20</td>
<td>&lt; 53</td>
<td>28.1</td>
<td>26.5</td>
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<tr>
<td>20</td>
<td>150</td>
<td>58.7</td>
<td>28.1</td>
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<td>&lt; 53</td>
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<td>150</td>
<td>81.8</td>
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Findings

- Dwell times for ALRVs are quite sensitive to:
  - Passenger flows
  - Passenger loads
- The crowding effect may well be non-linear.
- Dwell times for multi-car trains are different from those for one-car trains.
- The dwell time functions suggest high sensitivity of performance to perturbations
- Effective real-time operations control essential
- Running mixed train lengths dangerous
- Simulation models of high frequency, high ridership light rail lines need to include realistic dwell time functions.

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Heavy Rail Marginal Boarding Time

Heavy Rail Dwell Time Function

\[ DT = 12.22 + 2.27 \cdot B_d + 1.82 \cdot A_d + 6.2 \times 10^{-4} \cdot T_{Sd}^2 B_d \quad (R^2 = 0.89) \quad (9) \]

(12.82) \quad (7.11) \quad (9.07) \quad (4.70)

where

\[ A_d = \text{alighting passengers per door}, \]
\[ B_d = \text{boarding passengers per door}, \] and
\[ T_{Sd} = \text{through standees per door}, \]
i.e., total through standees divided by the number of doors