**Introduction to Analysis Approach**

- Basic approach is to establish an aggregate total cost function including:
  - operator cost as \( f \) (design parameters)
  - user cost as \( g \) (design parameters)
- Minimize total cost function to determine optimal design parameter (s.t. constraints)
- Variants include:
  - Maximize service quality s.t. budget constraint
  - Maximize consumer surplus s.t. budget constraint

**Bus Frequency Model: Square Root Rule**

- **Problem** define bus service frequency on a route as a function of ridership
- total cost = operator cost + user cost
  \[
  Z = c f(h) + b r h
  \]
  where
  - \( Z \) = total (operator + user) cost per unit time
  - \( c \) = operating cost per unit time
  - \( f \) = round trip time
  - \( h \) = headway – the decision variable to be determined
  - \( b \) = value of unit passenger waiting time
  - \( r \) = ridership per unit time

Minimizing \( Z \) w.r.t. \( h \) yields:

\[
 h = \frac{2cr}{br} \quad \text{or} \quad \frac{1}{2} \left( \frac{c}{h} \right) \left( \frac{t}{r} \right)
\]

This is the Square Root Rule with the following implications:
- High frequency is appropriate where (cost of wait time/cost of operations time) is high
- Frequency is proportional to the square root of ridership per unit time for routes of similar length
**Bus Frequency Model: Square Root Rule**

- Load factor is proportional to the square root of the product of ridership and route length

![Graph showing the relationship between bus capacity, ridership, and load factor.]

**Critical Assumptions**
- Bus capacity is never binding
- Wait time savings are the only benefits of higher frequency
- Ridership is linear
- Simple wait time model
- Budget constraint is not binding

**Possible Remedies**
- Introduce bus capacity constraint
- Modify objective function
- Introduce \( f(h) \) and re-define objective function
- Modify objective function
- Introduce budget constraint

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**Bus Frequency Example**

If

- \( c = \$90/\text{bus hour} \)
- \( b = \$10/\text{passenger hour} \)
- \( t = 90 \text{ mins} \)
- \( r = 1,000 \text{ passengers/hour} \)

Then

\[ h_{\text{OPT}} \approx 11 \text{ mins} \]

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**Bus Size Model**

**Problem** define optimal bus size for a route

**Assumptions**
- Desired load factor is constant
- Labor cost/bus hour is independent of bus size
- Bus dwell time costs per passenger are independent of bus size

**Using the same notation as before, plus**

- \( w = \text{labor cost per bus hour} \)
- \( p = \text{passenger flow past peak load point} \)
- \( k = \text{desired bus load (the decision variable)} \)

Then

\[ Z = w \cdot \frac{t}{h} + b \cdot r \cdot \frac{h}{2} \]

Now

\[ h = \frac{k}{p} \] by assumption above

\[ Z = \frac{wp}{k} + \frac{brk}{2p} \]

Minimizing \( Z \) w.r.t. \( k \) gives:

\[ k_{\text{OPT}} = \sqrt{\frac{2p^2w}{rb}} \]
Bus Size Model

- Result is another square root model, implying that optimal bus size increases with:
  - round trip time
  - ratio of labor cost to passenger wait time cost
  - peak passenger flow
  - concentration of passenger flows
- Previous example extended with:
  - $p = 500$ pass/hour
  - $w = $40/bus hour
  - all other parameters as before
- Then $k_{\text{OPT}} = 55$ passengers

Stop/Station Spacing Model

- Problem determine optimal stop or station spacing
  - Trade-off is between
    - walk access time (increases with station spacing)
    - in-vehicle time (decreases as station spacing increases)
    - operating cost (decreases as station spacing increases)
  - $Z = \text{total cost per unit distance along route and per headway}$
  - $T_{st} = \text{time lost by vehicle making a stop}$
  - $c = \text{vehicle operating cost per unit time}$
  - $s = \text{station/stop spacing} - \text{the decision variable to be determined}$
  - $N = \text{number of passengers on board vehicle}$
  - $v = \text{value of passenger in-vehicle time}$
  - $D = \text{demand density in passenger per unit route length per headway}$
  - $v_{\text{acc}} = \text{value of passenger access time}$
  - $w = \text{walk speed}$
  - $cs = \text{station/stop cost per headway}$

Stop/Station Spacing Model

\[
Z = \frac{T_{st}}{s} (c + N \cdot v) + c_s + \frac{s}{4} \cdot D \cdot \frac{v_{\text{acc}}}{w}
\]

Minimizing $Z$ w.r.t. $s$ gives:
\[
s_{\text{OPT}} = \left[ \frac{4w}{Dv_{\text{acc}}} \left( c_s + T_{st} (c_v + Nv) \right) \right]^{1/2}
\]

- Yet another square root relationship, implying that
  - station/stop spacing increases with:
    - walk speed $w$
    - station/stop cost $c_s$
    - time lost per stop $T_{st}$
    - vehicle operating cost $c_v$
    - number of passengers on board vehicle $N$
    - value of in-vehicle time $v$
  - and decreases with:
    - demand density $D$
    - value of access time $v_{\text{acc}}$

Bus Stop Spacing

U.S. Practice
- 200 m between stops (8 per mile)
- shelters are rare
- little or no schedule information

European Practice
- 320 m between stops (5 per mile)
- named & sheltered
- up to date schedule information
- scheduled time for every stop
Stop Spacing Tradeoffs

- Walking time
- Riding time
- Operating cost
- Ride quality

Walk Access: Block-Level Modeling

- Walking time
- Riding time
- Operating cost
- Ride quality

Example: MBTA Route 39

AM Peak Inbound results
- Average walking time up 40 s
- Average riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses

Bus Stop Locations and Policies

- Far-side (vs. near-side)
  - less queue interference
  - easier pull-in
  - fewer pedestrian conflicts
  - snowbank problem demands priority in maintenance

- Curb extensions
  - benefit transit, pedestrians, and traffic (0.9 min/mi speed increase)

- Pull-out priority
  - it's the law in some states

- Reducing dwell time
  - vehicle design
  - fare collection
  - fare policy