**Frequency Determination**

**Outline**
- Service Planning Hierarchy
- Introduction to Scheduling
- Setting Running Times and Cycle Times
- Frequency Determination

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**Service Planning Hierarchy**

<table>
<thead>
<tr>
<th>Planning Step</th>
<th>Frequency of Decisions</th>
<th>Principal Consideration</th>
<th>Principal Analysis Type</th>
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</thead>
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<td>Network Design</td>
<td>Infrequent</td>
<td>Service</td>
<td>Judgment &amp; Manual</td>
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<tr>
<td>Frequency Setting</td>
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<tr>
<td>Timetable Development</td>
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<td>Vehicle Scheduling</td>
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<tr>
<td>Crew Scheduling</td>
<td>Frequent</td>
<td>Cost</td>
<td>Computer-Based</td>
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**Introduction to Scheduling**

**Sequence of steps**

1. Determine running times and layovers based on
   - running time data
   - desired reliability levels
2. Determine frequencies by route and time period
3. Determine number of vehicles by time period
   - policies affecting integer constraints
   - revise step 1 and 2 decisions as needed
   - focus on transition periods
4. Determine timetable, typically
   - start at peak load point
   - generate start and end times
5. Chain vehicle trips together to form vehicle blocks
6. Cut and combine vehicle blocks to form crew runs

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**Service Planning Hierarchy**

- **Input**
  - Demand characteristics
  - Resources
  - Policies (e.g. coverage)

- **Function**
  - Network and Route Design
  - Frequency Setting
  - Timetable Development
  - Vehicle Scheduling
  - Crew Scheduling

- **Output**
  - Set of Routes
  - Service Frequency by Route, day, and time period
  - Departure/Arrival times for individual trips on each route
  - Revenue and Non-revenue Activities by Vehicle
  - Crew Duties
**Common Issues**

- **Integrality constraints**
  - If book times are 26 minutes each way, recovery time is 5 minutes at each terminus, and desired frequency is 10 per hour:
    \[ n_V = \frac{2 \cdot (26 + 5)}{6} = [10.3] = 11 \]
  - Trade-off between shortening cycle time by 2 minutes to save 1 vehicle, or not?
  - In a similar case, but if desired frequency is 1 per hour, choice is to:
    - shorten cycle time by 2 minutes, or
    - interline with another route having cycle time of 58 minutes or less

- **Marginal cost of additional trips**
  - A single trip for a vehicle/crew in peak period is typically uneconomic
    - eliminating the single trip and saving the vehicle/crew costs
    - adding additional trips to make a minimum sized "piece of work"
  - Where and when you add extra trips will affect costs.

- **Hard constraints**
  - Contract terms include hard/soft constraints which determine feasibility

**Setting Running Times and Cycle Times**

- **Key Input Data**
  - Actual running times
  - Current operations practices, e.g., time points

- **Typical Steps**
  - Define time points
  - Define time periods
  - For each time period:
    - set scheduled running time for full route and for each time point
    - set recovery time at end of trip

- **Example of Current Practice**
  - Use median running time for scheduled time
  - Set half-cycle time (scheduled time + minimum recovery time for 1-way trip) to 95th percentile of cumulative running time distribution

**Analysis of AVL Data Using Hastus ATP**

**Simple Rules and Current Practice**

- **Frequencies typically based on**
  - policy headways – vary by time of day and route type
  - maximum loads – vary by time of day and route type
  - These represent constraints rather than decision algorithms.

- **Maintain constant maximum load factor over periods**
  - at a level below official maximum load factor
  - may vary by time period

- **Maintain constant average occupancy level over periods**
  - subject to capacity constraint
  - may also be subject to a maximum time for loads above a specified level


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Importance of Frequency Determination

- Major short-range planning decision
  - Affects service quality through wait time and crowding
  - Affects transit path selection (assignment) in complex networks

- Two different contexts
  - Developed country city
    - ridership sensitive to service quality
    - sparse network, little transit path choice
    - maximum acceptable crowding levels specified
    - defined level of subsidy available
  - Developing country city
    - ridership constrained by capacity
    - crowding levels very high
    - dense network, significant transit path choice

Maximize Social Surplus (multiple routes problem)

Context

- Given a fixed fleet size and subsidy,
- Determine optimal allocation of this fleet to the various routes (thus setting the frequencies on the routes)

Formulation

- Maximize social surplus across all routes
- Subject to
  - subsidy not exceeded
  - fleet size not exceeded
  - level of service is acceptable (meets service delivery policy)

Developed Country Frequency Determination Problem

- Decision variables
  - headway on each route for each time period
- Objective function
  - maximize consumer surplus + social ridership benefit
    - \((b \times \text{wait time savings}) + (a \times \text{ridership})\)
- Constraints
  - total subsidy is exhausted
  - total fleet size is not exceeded
  - headway meets policy maximums and loading maximums


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Maximize Social Surplus

Social Surplus

1. Consumer Surplus
   - Recall that waiting time is a function of headway

\[
\text{ridership} \quad r(w) \\
\text{consumer surplus} \\
\text{demand function} \\
\text{wait time savings for } r_i^{th} \text{ rider}
\]
Maximize Social Surplus

- For a given headway $h^*$, $w^* = f(h^*)$
- Consumer surplus is

$$CS = b \int_{w^*}^{\infty} r(w)dw$$

where

$b$ = monetary value of waiting time

$CS$ = savings in wait time cost that accrues to riders

who would have been prepared to ride at higher

waiting times

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Maximize Social Surplus

- Since $w = f(h)$, we can derive $r(h)$ from $r(w)$, i.e.

$$r(h) = r(f(h))$$

Total social surplus to maximize:

$$CS + SB = \sum_{\text{routes}} \left[ b \int_{h_i^*}^{\infty} r(h)dh + ar(h_i^*) \right]$$

where $h_i^*$ is the headway on route whose optimal value is to be determined (decision variable)

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Maximize Social Surplus

2. Social Benefits (of transit)

- mobility for non-auto owners
- reduced congestion
- reduced pollution
- reduced energy consumption
- positive land use effects

- All of these benefits are highly associated with ridership
  - Social benefit for a route $= a \cdot r(w)$
  - where $a$ = monetary value of social benefit associated with an additional rider less the fare

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Maximize Social Surplus: Constraints

Subsidy

$$\sum_{\text{routes}} [\text{operating cost} - \text{fare revenue}] = \text{subsidy limit}$$

$$\sum_{\text{routes}} [c(h_i^*) - F \cdot r(h_i^*)] = S_o$$

Fleet Size

$$\sum_{\text{routes}} \frac{\text{round-trip time}}{h_i^*} \leq \text{Fleet size, } M$$

Level of Service

$$h_i^* < h_0$$ headway standard

$$g(h_i^*) < l_0$$ load standard

vehicle load
Maximize Social Surplus

Critical Assumptions and Limitations
- independence across routes
  - In model, ridership on a route depends only on the headway of that route.
  - In reality, ridership also depends on headways on competing routes and complementary routes (transfers).
- network design is not considered

Advantages
- ridership = \( f(\text{frequency}) \)
- captures trade-offs across routes
- introduces system wide budget constraint

- Square root rule is valid where constraints are not binding
- Problem can be solved using lagrangian relaxation and single variable search techniques (not very complex)
- Existing scheduling practice over allocates service to peak and to long, high ridership routes
- Minimizing wait time assuming fixed demand gives similar solutions to more complex objective and variable demand
- Best allocation of resources is quite robust with respect to objectives and parameters assumed

Efficiency in Subsidy Allocation
This is a resource allocation problem.
For optimality, allocate enough resources to each route so that Marginal Benefit/Cost Ratio is same on each route.

Developing Country City Frequency Determination Problem

Objectives
- minimize crowding levels
- minimize waiting times

Constraints
- loading feasibility (vehicle capacity)
- passenger assignment
- total fleet size
Passenger Assignment Heuristic Approach

1. Classify flow into:
   a. captive flow (CF) - any OD pairs with only one feasible path
   b. variable flow (VF) - OD pairs with more than one feasible path

2. Assign VF in proportion to frequency share on acceptable routes, consistent with random bus arrival process

\[
\frac{D_i}{\sum_{j \in J} D_j} = \frac{F_i}{\sum_{j \in J} F_j}
\]

where
- \(D_i\) = demand assigned to route \(i\) for specific OD pair
- \(F_i\) = frequency offered on route \(i\)
- \(J\) = set of acceptable routes

Models

- Normative (Ideal) Model
  - assign passenger flows to routes with minimum round trip vehicle time among all acceptable paths
  - compute frequency and fleet size required on this assignment basis

- Descriptive (Realistic) Model
  - assign passengers to alternative acceptable paths in proportion to frequency share in an iterative process

- The difference in the total fleet sizes from the normative and descriptive models indicates the extent of inefficiency resulting from the overlapping route structure.

Simple Example of Overlapping Routes

- OD pair \(cd\) is VF, all other pairs are CF
- Ideally, \(cd\) flow would be assigned to route 1, which is shorter, but in reality these passengers will take route 1 or 2, whichever arrives first.
- Some \(ce\) passengers may be forced to board route 1 buses, then make a transfer at \(d\) to route 2.